# Week 10: High Frequency Behaviour, Positive Feedback, and Oscillators

### 10.1 More on RC filters

We saw that the *high-pass RC filter* has a gain of less than 1 which is asymptotically proportional to the signal frequency below the 3dB roll-off point. The circuit also introduces a phase shift to the signal at low frequencies which asymptotically approaches +90°: that is, the output signal voltage leads the input signal in phase. At low frequency the reactance of $C$ is larger than $R$ so:

$$i \to C\frac{\mathrm{d}v_{\mathrm{in}}}{\mathrm{d}t}$$

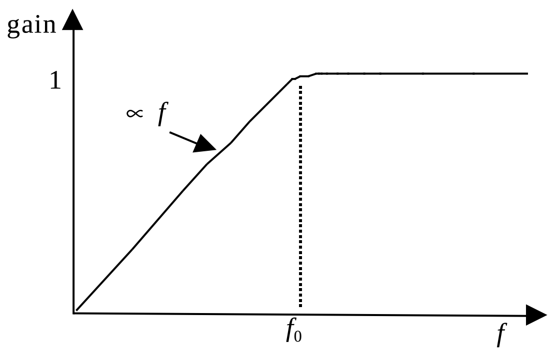
$$v_{\mathrm{out}} = Ri \to RC\frac{\mathrm{d}v_{\mathrm{in}}}{\mathrm{d}t},$$

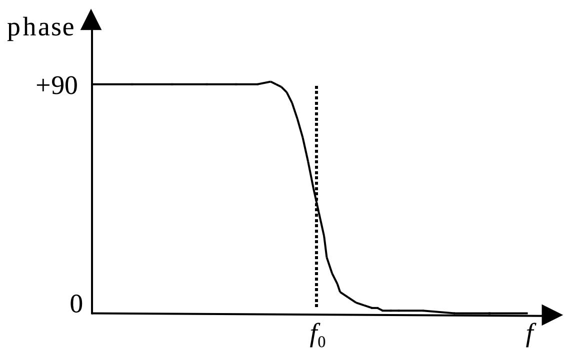
$$\text{if } \; v_{\mathrm{in}} = \sin\omega t \quad \text{then}$$

$$v_{\mathrm{out}} = RC\omega\cos\omega t = RC\omega\sin(\omega t + 90^{\mathrm{o}}).$$

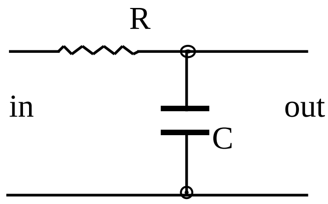
If we define $\omega_0 = 1/(RC)$ then

$$gain = \frac{|v_{\mathrm{out}}|}{|v_{\mathrm{in}}|} = \frac{\omega}{\omega_0} = \frac{f}{f_0} \quad , \quad \text{for } \; f < f_0 = \omega_0/2\pi$$



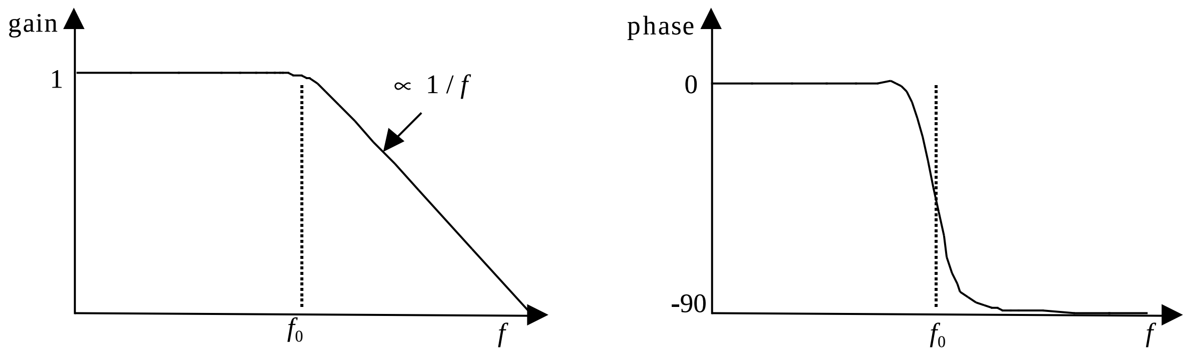




The circuit of the single-pole *low-pass RC filter* is




This has unity gain and zero phase shift at low frequency but at frequencies above the 3dB roll-off point the gain becomes asymptotically inversely proportional to the frequency and the phase shift approaches –90°: the output lags the input signal.

$$f_{\mathrm{o}} = \frac{1}{2\pi RC}, \quad \text{gain} \to \frac{f_{\mathrm{o}}}{f}.$$




Any circuit starts to behave as a low-pass filter at sufficiently high frequency, and at high enough frequency the rate of fall-off in gain and the phase shift generally become larger than for a single-pole filter.

## 10.2 Bandwidth

The range of frequency over which a circuit has constant gain to within 3dB is called its *bandwidth*. For a DC coupled circuit this will usually be the same as the high frequency roll-off point, $f_2$. For an AC coupled circuit it is the difference between the low frequency and the high frequency roll-off points: $f_2 - f_1$.

For a negative feedback amplifier with a simple resistive feedback network the bandwidth of the feedback network is invariably very much larger than that of the amplifying element. Since the closed-loop gain is determined by the feedback factor this means that the closed-loop gain will have a larger bandwidth than the open-loop gain.

The high frequency 3dB roll-off point $f_2$ of the closed-loop gain is determined from the equation (using complex notation to include phase shifts and taking β and $A_{0f}$ to be always real):

$$\frac{A_{of}}{\sqrt{2}} = \left|\frac{A}{1+\beta A}\right|$$

$$\left|\frac{1}{A}+\beta\right| = \frac{\sqrt{2}}{A_{of}}$$

$$\left|\frac{1}{A}+\frac{1}{A_{of}}\right| \approx \frac{\sqrt{2}}{A_{of}}$$

$$\left|\frac{A_{of}}{A}+1\right| \approx \sqrt{2}$$

$$\frac{A_{of}^2}{|A|^2}+1 \approx 2$$

$$\frac{A_{of}^2}{|A|^2} \approx 1$$

$$|A(f_2)| \approx A_{of},$$

where $A_{of}$ is the low frequency closed-loop gain. Note that $A$ has been assumed to be purely imaginary at high frequency (phase shift = –90°).

If the op amp has a gain-bandwidth product of $f_B$ (unity gain frequency) then at high frequency $|A| = f_B/f$, so the closed-loop high frequency roll-off point is simply given by

$$f_2 = f_B / A_{of}\,.$$

## 10.3 Instability with NFB

The negative feedback equation is

$$S_o = \left(\frac{A}{1+\beta A}\right) S_s\,.$$

We can therefore see that a phase shift in β and/or $A$ can lead to the magnitude of the denominator becoming less than 1 and the closed-loop gain will then be greater than the open-loop gain. Under these circumstances all of the advantages of negative feedback are lost.

More serious is the possibility that the denominator becomes equal to zero when $\beta A = -1$. The closed-loop gain is then infinite and we have an output for zero input: the circuit will spontaneously oscillate at the frequency and with an amplitude which correspond to that condition. This condition is called *positive feedback.*

Positive feedback is <u>required</u> for an *oscillator* circuit. We must ensure that the oscillation frequency and amplitude are well determined by circuit components. However, positive feedback is *undesirable* for an amplifier since the oscillation would swamp the input signal and the circuit would be useless.

Since high frequency phase shift is unavoidable it is necessary to take precautions to ensure that the amplifier is far from the positive feedback condition at all frequencies. The easiest way to do this is to apply the *Bode stability criteria*:

1 At the frequencies where $|\beta A| = 1$, the open-loop phase shift (combined $\beta A$ phase shift) should be less than 180–45 = 135° (positive or negative);
2 At the frequencies where the open-loop phase shift is ±180°, the value of $|\beta A|$ should be less than ⅓ (–10dB).

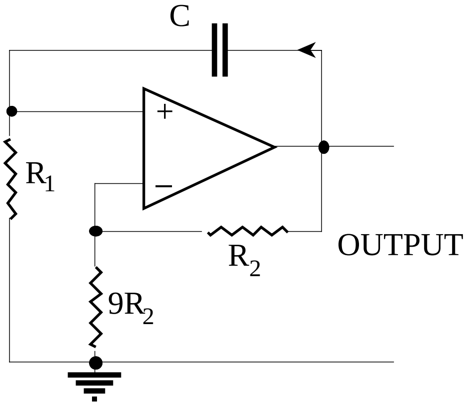
Most commercially available op amps contain internal component to ensure that the Bode conditions are satisfies even when β=1. Generally speaking, the closer β is to 1 the bigger the risk of instability.

The virtual earth differentiator circuit of last week is an example of a circuit which fails the Bode conditions. The feedback network is a low pass filter with a phase shift of –90°. The op amp at high frequency will also have a phase shift approaching –90°. The combined open-loop phase shift will therefore easily approach –180° and there is a serious risk that the circuit will oscillate. So this is a further reason why an extra small resistor and/or capacitor must be included in the feedback circuit so that at high frequency the open-loop phase shift is limited to –90° or brought back close to 0°.

**Exercise 10.1:**
Using Crocodile Clips construct the virtual earth differentiator with a time constant of 1ms and verify its operation.
Now simulate a degraded high frequency behaviour for the op amp by placing a low-pass RC filter at the output terminal of the op amp and adjust the time constant until the circuit becomes unstable. If one low-pass filter is ineffective put in two of equal time constants.
Estimate how close your circuit comes to the Bode stability conditions.

### 10.4 The multivibrator

The simplest form of oscillator using an op amp is the multivibrator where the output is fed back to the *non-inverting* input (positive feedback) through a high-pass RC filter:




If we think of the output voltage as being at its most positive possible, $+V_{max}$, and the potential at both the op amp's input terminals as being close to $+0.9V_{max}$, then the capacitor is charging up and the potential at the non-inverting input is falling.

When it falls to just below the potential at the inverting input, $+0.9V_{max}$, the op amp output will be driven to its most negative possible potential, $-V_{max}$, and this will result in the inverting input going to $-0.9V_{max}$, and the non-inverting input going to $+0.9V_{max} - 2V_{max} = -1.1V_{max}$, which will therefore maintain the op amp output in its "low" state.

The capacitor now charges in the reverse direction and the voltage at the non-inverting input rises towards zero. When it reaches $-0.9V_{max}$ the op amp will flip back to its "high" state. This cycle will repeat indefinitely with the output flipping between the extreme values at a frequency determined by the time constant *RC* and the ratio of the resistors in the inverting input feedback circuit.

**Exercise 10.2:**
Using Crocodile Clips design a multivibrator that oscillates at a frequency of 100Hz. Determine the values of $R$ and $C$ by choosing any values, measuring the oscillation frequency, and then adjusting them in proportion to give the desired frequency. What is the product of the oscillation frequency and the $RC$ time constant?

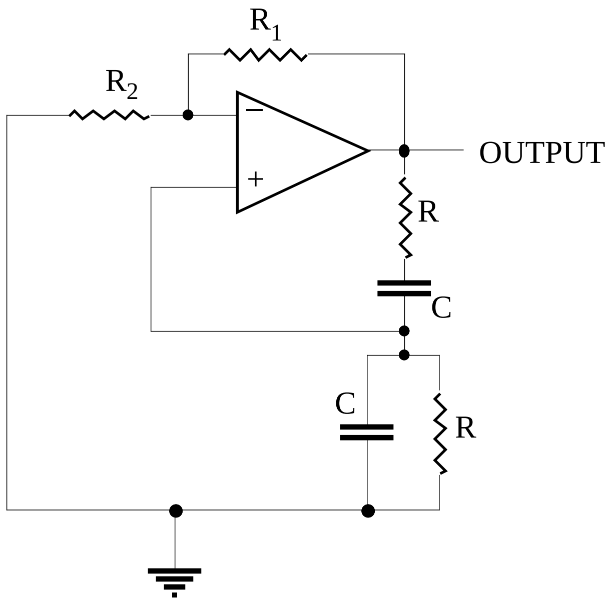
If you have time, build the circuit using the op amp board and compare its performance.

Warning. The potential at the non-inverting input swings between values slightly greater than the maximum output voltage. If this is greater than the supply voltage, the op amp may be destroyed. If this happens, reduce the supply voltage values or increase the ratio of the resistors in the inverting input feedback circuit. If you do the latter, the oscillation frequency will change.

### 10.5 The sine wave oscillator

The output of the multivibrator is a square wave and the frequency is not precisely and simply relate to an $RC$ time constant. To obtain a sine wave output whose frequency is precisely determined by an $RC$ time constant we can use a feedback circuit which has zero phase shift at only one frequency and connect this to the non-inverting input of the op amp. This is called the Wien oscillator.

If the inverting input were connected directly to earth then the op amp would be overdriven and a distorted output wave form would result. We therefore include some negative feedback to the inverting input which is just slightly less than the positive feedback to the non-inverting input, which keeps the differential input voltage within bounds.




The circuit is now called a Wien Bridge oscillator. The oscillation conditions are

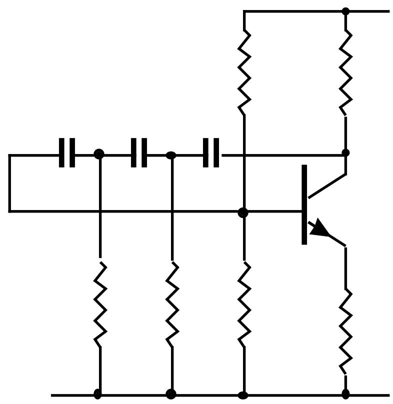
$$f = \frac{1}{2\pi RC},$$

$$R_1 \geq 2R_2.$$

**Exercise 10.3:**
Construct a Wien Bridge oscillator to have a frequency of 100Hz. Include a small variable resistor in the NFB circuit so that the NFB factor can be adjusted. Observe the effect on the output wave form.
[Note: with positive feedback circuits Crocodile Clips can take some time to build up an oscillation.]

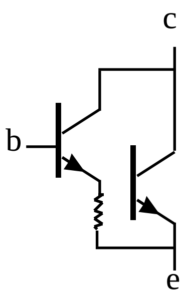
### 10.6 The "phase shift" oscillator

A simple sine wave oscillator can be constructed using a single transistor, provided it's $\beta$ is large enough (> 200). The basic common-emitter amplifier circuit is set up with the addition of three successive equal high-pass *RC* filters connected from the collector to the base. The common-emitter amplifier is an inverting amplifier so direct connection from collector to base would give negative feedback. The three high-pass filters combine to give an overall phase shift of 180° so we then have positive feedback and the circuit will oscillate.



**Exercise 10.4:**
Construct a transistor phase-shift oscillator to oscillate at 100Hz. What is the relationship between the filter time constant and the oscillator frequency?

Note: the common-emitter circuit must have a gain of at least 30 for this circuit to work. Use the highest possible supply voltage in order to achieve this. If you still can't make it work then try a pair of transistors connected in cascade in place of the single one:




[This arrangement is called a "super-$\beta$ pair". Choose the resistor to be around 10 kΩ.]