Week 11: Analogue Computer Circuits

11.1 Using an op amp to solve an equation

When an op amp is part of a circuit with feedback and its output is not saturated (i.e. of a magnitude less than the maximum possible) then the two inputs of the op amp are at essentially the same potential. If these two potentials are functions of potentials at other parts of the circuit, and perhaps also of time, then the circuit is in effect settling at the solution to the equation relating those two functions. We can look at this situation the other way round, starting with an equation we would like to solve and then designing a feedback circuit whose potentials will be the solution to the equation.

The most straightforward way to implement such an “analogue computer” circuit is to build it around the analogue summer. We arrange the equation with one term as its subject, the left hand side, and all the other terms on the RHS. If there are N terms on the RHS then we need a summer with N inputs. The output of the (inverting) summer is obviously then (minus) the term on the LHS of the equation. It is then a matter of linking the summer output to its various inputs using circuits that correspond to the functional relationships between these terms.

11.2 Realising the terms of equations

In physics and engineering most equations of interest are differential equations. We therefore need a means of making one signal the time differential of another. The differentiator op amp circuit has various practical difficulties associated with it, as explained last week, so we actually use *integrators.* The output of an integrator is the integral of the input, so the input is the differential of the output. Since the circuit is to be used in a closed loop which will settle down to an equilibrium, it makes no difference whether a particular value is an input or an output to a particular op amp, since it will be the output or input, respectively, of the adjacent op amps.

The fact that we are using only integrators to represent differential equations does have one practical consequence: **the output of the summer must be the highest order derivative used in the equation.** We should begin, therefore, by arranging the equation so as to have only the highest order derivative on the LHS.

The other algebraic operations that are required are obviously addition and subtraction and multiplication and division. Addition is already covered since we are beginning with a summer circuit. Subtraction is obtained by placing a unity gain inverter before one of the inputs to the summer.

Multiplication or division by a *constant* is easily arranged using a simple amplifier or potential divider which can have variable resistors included so that we can adjust the values of the constants.

Multiplication or division of two variables in the equation, i.e. of two potentials in the circuit, requires something new. This is done using a logarithmic amplifier which we will look at later.

It is implicit in this analysis that the equation to be solved has at most one independent variable which will be represented by *time.*

A *non-homogeneous* equation will have a term which is not functionally related to any of the others, such as $\sin\omega t$. Such terms are included in the circuit as inputs from external sources: a sine-wave oscillator in this instance.

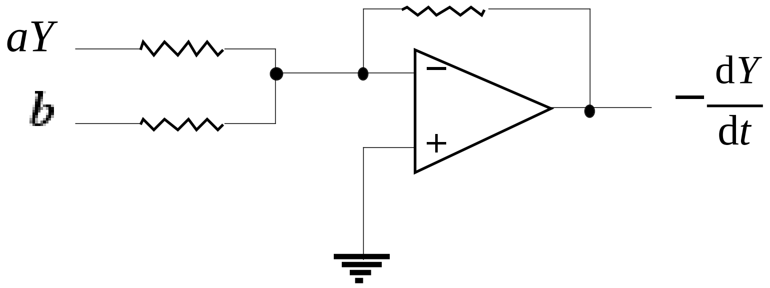
11.3 A simple example: first order linear differential equation

The most general form of such an equation can be written as

$$\frac{dY}{dt} = aY + b$$

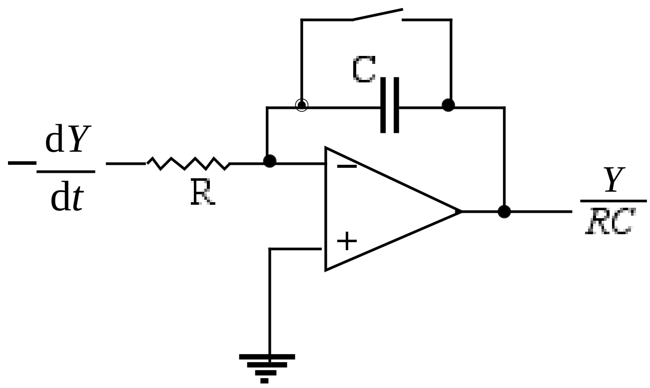
Notice how the differential term is made the subject of the equation. Y is the dependent variable and a and b are of course constants.

We begin by setting up the summer circuit, labelling its two inputs and output according to our equation:

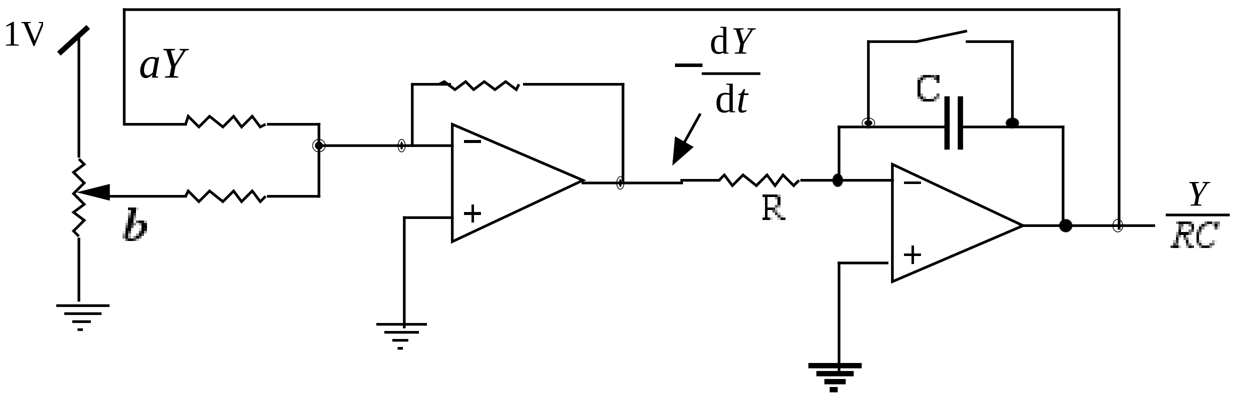


The constant term b is not related to any other term, it is a fixed potential. This can therefore be obtained from the power supply via an appropriate potential divider to give the value required.

The term aY is related to the output of the summer through an (inverting) integrator, and the value of the constant a is determined by the time constant of the integrator, $a = 1/(RC)$:



In practice the summer is an *inverting* circuit, but so is the integrator so these two inversions cancel out. So now we can connect up the entire circuit:



Note that the "solution" to the equation, Y, doesn't actually appear anywhere. The nearest to it is Y/RC, so if we want exactly Y we would need to include an additional amplifier or potential divider to scale this signal to give our final output.

We can see that the overall feedback is *positive* (the closed loop phase shift is –90°), so we would expect that the solution will not be stabile. In fact it increases exponentially until one of the op amps saturates. It is important therefore to include the switch in order to establish the initial conditions: $Y = 0$ at $t = 0$.

Exercise 11.1:
Modify the above circuit so that the constant a is a negative number. This should now produce an asymptotically stable solution.
Construct the circuit in Crocodile Clips, deriving b from a square wave oscillator so as to easily observe the effect of alternating positive and negative values.

11.4 More complicated example: the driven damped oscillator

Now we have an inhomogeneous linear second order differential equation which we can write in its general form as

$$\frac{\mathrm{d}^2Y}{\mathrm{d}t^2} = a\frac{\mathrm{d}Y}{\mathrm{d}t} + bY + c + d\sin\omega t$$

The driving term is taken to be a harmonic wave, $d\sin\omega t$ plus an offset c.

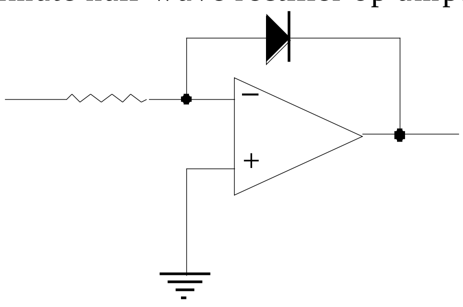
Clearly, to relate the second order derivative to the first order derivative and to the linear term bY we will need <u>two</u> consecutive integrators. a will be given by the time constant of the first of these and b will be given by the product of the two time constants.

Exercise 11.2:
In Crocodile Clips construct the circuit to solve this equation. Initially, choose the signs of the constants a and b so as not to require the inclusion of extra unity gain inverters.
Try out the circuit to see if it really is a driven damped oscillator. If you feel that the sign of a and / or b needs to be changed, insert the appropriate inverters.
Determine the relationships between the constants a and b and the resistors and capacitors in the two integrators.

Exercise 11.3:
Use the analogue computing board to construct a circuit to solve a second order differential equation. Only two variations are possible using this board: try them both and deduce the corresponding equations and solutions.

11.5 Analogue multiplier

The basic circuit of the logarithmic amplifier is the same as what we previously considered to be an approximate half-wave rectifier op amp:



We need to use it only in the forward biassed region, i.e. positive input for the circuit shown.

As it stands this circuit is not very useful as a logarithmic amplifier for several reasons.

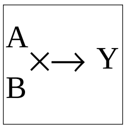
1. The slope resistance of the diode is very small so the output voltage is very small: so small that we considered it to be negligible when regarding the circuit as a rectifier.
2. A standard diode breaks away from the ideal logarithmic law at very small and very high values of current.
3. The gain of the circuit depends strongly on temperature because the diode characteristic is temperature dependent.

To correct all of these defects requires quite a complicated circuit and a transistor base-emitter junction is used instead of the simple diode. All of this is available packaged together as a single integrated circuit.

Even as a carefully designed integrated circuit the logarithmic amplifier still needs many more components to create a full multiplier. We need two logarithmic amplifiers to multiply two numbers. We need a summer to combine the two logarithms. Then we need an antilog circuit to generate the final result. This is done by interchanging the resistor with the diode in the above circuit.

Even now, our multiplier will only work for positive input values. A full 4-quadrant analogue multiplier is therefore an extremely complex circuit, but it too is available as a single integrated circuit.

Exercise 11.4:
Crocodile Clips does not include an analogue multiplier (and it would be an impossibly complicated task to build one up from the available elements).
Using the symbol



to represent the circuit that provides the function $Y = A \times B$, draw the circuit to find the square of a voltage. (Yes, this IS very simple.)

Now draw an op amp circuit that will find the square root of a voltage.
Hint: to do this you will now need to think of the feedback "factor" β as a function, not just as a simple multiplier. Rework the equations for the non-inverting amplifier with this change to the notation when you will see that the overall op amp *transfer function* is the *inverse* of the feedback *function*.

Exercise 11.5:
Draw the circuit to solve the non-linear equation:

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = aY^2 + bY + c.$$

Exercise 11.6:
Using the simple diode logarithmic amplifier and the equivalent antilog amplifier, construct in Crocodile Clips a positive-input squaring circuit and determine its accuracy for a range of values. Chose resistor values to give a diode current of a few milliamps.

Note: You should not expect your circuit to produce an output of 1V for an input of 1V. We could express its operation by the equation

$$V_{out} = aV_{in}^2$$

where the constant *a* will depend on the precise characteristics of the diode and should be found empirically. You can try placing a 2-input summer before the antilog circuit so as to add in a constant voltage to bring the value of *a* to 1.