3C43

LASERS
&
MODERN OPTICS

## 2 Lasers

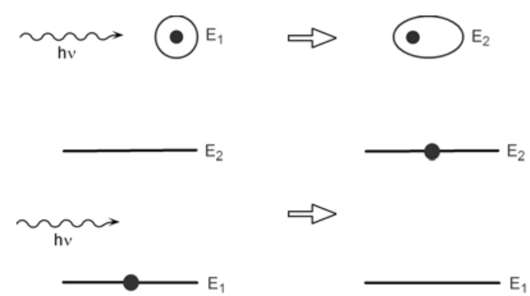
2.1 Principles of laser action

## The emission and absorption of radiation

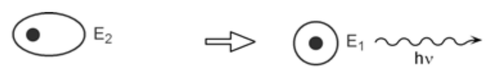
Einstein proposed that three types of fundamental interaction were necessary to understand the interaction of light and matter under conditions of thermal equilibrium.

- Absorption

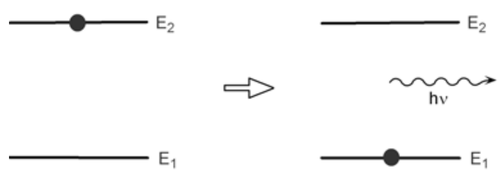
$h\nu \approx E_2 - E_1$



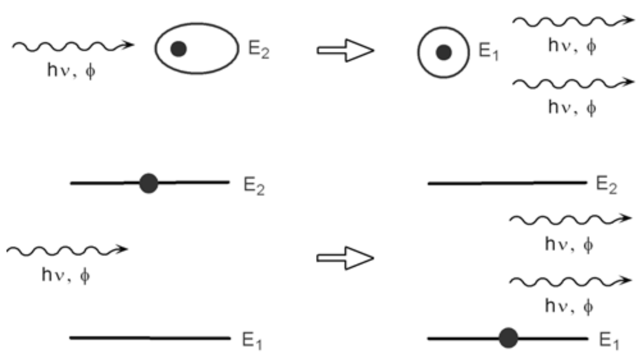
- Spontaneous emission




$$h\nu = E_2 - E_1$$

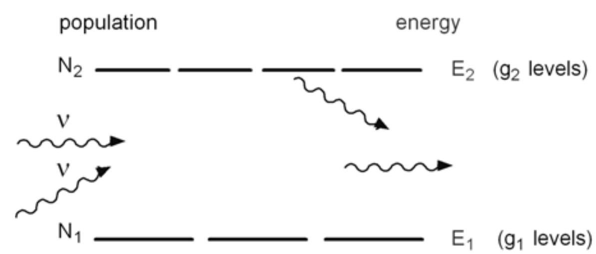



- Stimulated emission




## Einstein A and B coefficients

- Atom with a ground state and single excited state (which may be degenerate) interacting with light at frequency ν




$$\left(\frac{dN_2}{dt}\right)_{spont} = -A_{21}\, N_2$$

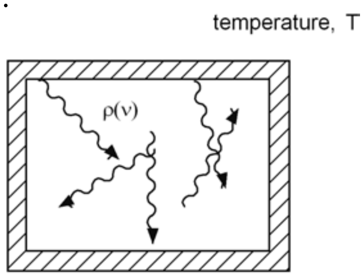
$$\left(\frac{dN_2}{dt}\right)_{stim} = -B_{21}\, N_2\, \rho(\nu)$$

$$\left(\frac{dN_1}{dt}\right)_{abs} = -B_{12}\, N_1\, \rho(\nu)$$

## Einstein relations

• Simplified model: zero transition linewidth

• Einstein considered the case of thermal equilibrium between a cavity at temperature T and the radiation in the cavity.




• He considered the cavity walls to be made up of a set of non-interacting, 2-level atoms.

• The intra-cavity radiation contains many frequency components, but the spectral energy density at any particular wavelength is constant.

## Einstein relations

$$\left(\frac{dN_2}{dt}\right)_{TOTAL} = 0 = -N_2 A_{21} - N_2 B_{21}\,\rho(\nu_0) + N_1 B_{12}\,\rho(\nu_0)$$

◆ $$\rho(\nu_0) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$= \frac{A_{21}}{B_{12}\left(N_1 / N_2\right) - B_{21}}$$

In the case of thermal equilibrium

. $$\frac{N_1}{N_2} = \frac{g_1}{g_2}\exp\left(\frac{h\nu_0}{k_B T}\right)$$

. $$\rho(\nu_0) = \frac{8\pi h\nu_0^3}{c^3}\frac{1}{\exp\left(\frac{h\nu_0}{k_B T}\right) - 1}$$

◆ $$\frac{A_{21}}{B_{12}\left(\frac{g_1}{g_2}\right)e^{\frac{h\nu_0}{k_B T}} - B_{21}} = \frac{8\pi h\nu_0^3}{c^3}\frac{1}{e^{\frac{h\nu_0}{k_B T}} - 1}$$

◆ $$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu_0^3}{c^3}, \quad g_2 B_{21} = g_1 B_{12} \quad !$$

Example

Find the ratio of the spontaneous emission rate to the rate of stimulated emission for the case of a tungsten filament lamp operating at a temperature of 1800K if the peak of the lamp's emission spectrum corresponds to green light.

Solution

The required ratio, $R = \dfrac{A_{21}}{\rho(\nu_0)B_{21}}$

Using $\rho(\nu_0) = \dfrac{8\pi\, h\nu_0^3}{c^3}\dfrac{1}{e^{\frac{h\nu_0}{k_BT}} - 1}$

and $\dfrac{A_{21}}{B_{21}} = \dfrac{8\pi h\nu_0^3}{c^3}$

gives $R = \exp\left(\dfrac{h\nu_0}{k_BT}\right) - 1$

Here $R = \exp\left(\dfrac{6.63\times10^{-34}\times(3.00\times10^8/530\times10^{-9})}{1.38\times10^{-23}\times1800}\right) - 1$

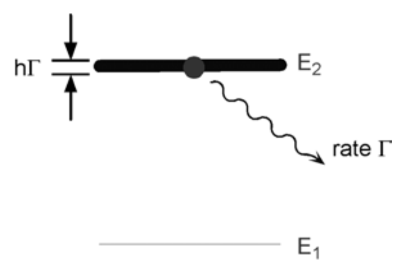
$R \approx 3.6\times10^6$

## The atomic lineshape function

• A sample of atoms will usually absorb and emit light over a finite range of frequencies as a result, for example, of:

- the natural linewidth of the atomic transition,
- Doppler broadening,
- collisions between atoms

• e.g. Broadening resulting from the natural lifetime of the atomic excited-state




## The linewidth of an atomic transition

• Homogeneous broadening:

- All atoms in the sample make the same contribution to the response at all frequencies;
- Lorentzian lineshape function

e.g.

- Natural linewidth
- Collisional (pressure) broadening

• Inhomogeneous broadening

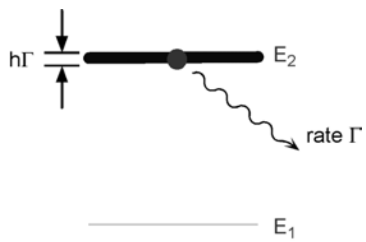
- The atoms in the sample contribute differently to different frequency components of the total response;
- Gaussian lineshape function

e.g.

- Doppler broadening
- Broadening due to spatial variation in the environment (magnetic/electric field, the presence of localized perturbers)

- Homogeneous broadening

e.g. Natural lifetime broadening




In a classical model, the amplitude of the emitted, damped EM-wave can be represented by

$$a(t) = \exp i \cdot 2\pi\nu_0 t \times \exp -\frac{\Gamma t}{2}$$

Taking a Fourier transform

$$\tilde{a}(\nu) = \int_0^\infty \exp - i(2\pi\nu t) \cdot \exp i(2\pi\nu_0 t) \cdot \exp -\frac{\Gamma t}{2} \cdot dt$$

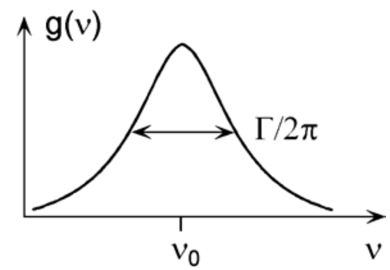
So $\tilde{a}(\nu) \propto \dfrac{1}{2\pi i(\nu - \nu_0) - \Gamma/2}$

Then $g(\nu) \propto \left|\tilde{a}(\nu)\right|^2 \propto \dfrac{1}{(\nu - \nu_0)^2 + \left(\Gamma/4\pi\right)^2}$

a Lorentzian lineshape

Normalizing: $\int g(\nu) \cdot d\nu = 1$ gives

$$g(\nu) = \frac{\Gamma/4\pi^2}{(\nu - \nu_0)^2 + \left(\Gamma/4\pi\right)^2}$$



• Inhomogeneous broadening

Different classes of atoms contribute to different parts of the sample response.

eg. Doppler broadening

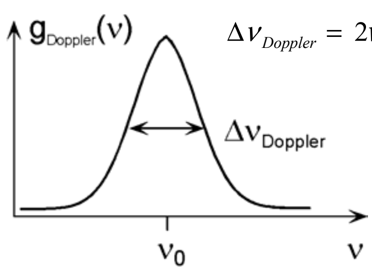
Using $p(\mathrm{v}_x) = \sqrt{\dfrac{m}{2\pi k_B T}}\ \exp-\left(\dfrac{m\mathrm{v}_x^2}{2k_B T}\right)$

and $\nu - \nu_0 = \nu_0 \dfrac{\mathrm{v}_x}{c}$

gives $g_{Doppler}(\nu) = \dfrac{c}{\nu_0}\sqrt{\dfrac{m}{2\pi\, k_B T}}\ \exp-\left(\dfrac{mc^2}{2k_B T}\right)\left[1-\dfrac{\nu}{\nu_0}\right]^2$

and full- width at 1/e maximum is

$$\Delta\nu_{Doppler} = 2\nu_0\sqrt{\frac{2\,k_B\,T}{mc^2}}$$




Summary of useful results

• Homogeneous case

$$g(\nu) = \frac{\Gamma/4\pi^2}{(\nu-\nu_0)^2 + \left(\Gamma/4\pi\right)^2}$$

·Full-width at half maximum

$$\Delta\nu_{1/2} = \frac{\Gamma}{2\pi}$$

·Line-centre value

$$g(\nu_0) = \frac{2}{\pi\,\Delta\nu_{1/2}}$$

• Inhomogeneous case

$$g_{Doppler}(\nu) = \frac{c}{\nu_0}\sqrt{\frac{m}{2\pi\, k_B T}}\ \exp -\left(\frac{mc^2}{2k_B T}\right)\left[1-\frac{\nu}{\nu_0}\right]^2$$

·Full-width at 1/e maximum

$$\Delta\nu_{Doppler} = 2\nu_0\sqrt{\frac{2\, k_B\, T}{mc^2}}$$

·Line-centre value

$$g(\nu_0) = \frac{2}{\sqrt{\pi}\,\Delta\nu_{Doppler}}$$

• In both homogeneous and inhomogeneous cases, it is often a good approximation to write

$$g(\nu_0) = \frac{1}{\Delta\nu}$$

## The absorption coefficient

• To characterize the effect of finite atomic linewidth, introduce the lineshape function $g(\nu)$

• $g(\nu)$ is a function peaked around

$\nu = \nu_0 = \frac{E_2 - E_1}{h}$ and $\int g(\nu)\cdot d\nu = 1$

Summing over all wavelengths:

$$\left(\frac{dN_2}{dt}\right)_{stim} = -\int g(\nu)\, B_{21}\, N_2\, \rho(\nu)\cdot d\nu$$

$$\left(\frac{dN_1}{dt}\right)_{abs} = -\int g(\nu)\, B_{12}\, N_1\, \rho(\nu)\cdot d\nu$$

With $g(\nu) = \delta(\nu - \nu_0)$ (Kronecker delta)

we recover $\left(\frac{dN_2}{dt}\right)_{stim} = -B_{21}\, N_2\, \rho(\nu_0)$

$\left(\frac{dN_1}{dt}\right)_{abs} = -B_{12}\, N_1\, \rho(\nu_0)$

Let N = number per unit volume of photons
$N_\nu$ = number per unit volume of photons per unit frequency at frequency $\nu$,
$N_1$, $N_2$ = number per unit volume of atoms in ground and excited states

$$\frac{dN}{dt} = +\int g(\nu)\left(B_{21}N_2 - B_{12}N_1\right)\rho(\nu)\cdot d\nu$$

$$= \int \frac{dN_\nu}{dt}\cdot d\nu$$

where $\frac{dN_\nu}{dt} = +g(\nu)\left(B_{21}N_2 - B_{12}N_1\right)\rho(\nu)$

$$= -g(\nu)\left(\frac{g_2}{g_1}N_1 - N_2\right)B_{21}\rho(\nu)$$

Using the spectral irradiance $I_\nu = \rho(\nu)\, c/n$
and the identity

$$\rho(\nu) = N_\nu h\nu$$

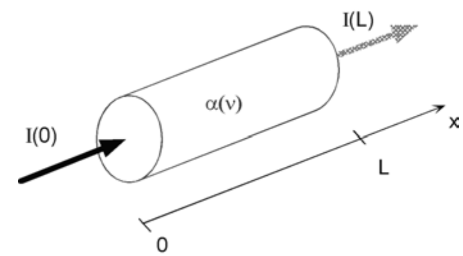
gives

$$\frac{dI_\nu}{dx} = \frac{n}{c}\frac{dI_\nu}{dt} = -g(\nu)\left(\frac{g_2}{g_1}N_1 - N_2\right)B_{21}\frac{nh\nu}{c}\cdot I_\nu$$

So $\frac{dI_\nu}{dx} = -\alpha(\nu)\cdot I_\nu$

where $\alpha(\nu) \equiv g(\nu)\left(\frac{g_2}{g_1}N_1 - N_2\right)B_{21}\frac{nh\nu}{c}$ !

is the small-signal absorption coefficient for incident light of frequency $\nu$.




Near resonance, incident monochromatic light is exponentially attenuated (Beer's law)

$$I_\nu(L) = I_\nu(0)\cdot\exp(-\alpha(\nu)\,L) \quad !$$

(provided $N_2 - \frac{g_2}{g_1}N_1 < 0$ )

## The spectrum of absorbed light

Note that for incident broad-bandwidth light, the spectrum of the absorbed light is generally has a shape which differs from $g(\nu)$

Intensity transmitted at frequency $\nu$:

$$I_\nu(L) = I_\nu(0) \cdot \exp(-\alpha(\nu)\, L)$$

Close to zero over a large range of $\nu$

This is also observed when $\alpha$<0 (so there is gain) and is one reason why laser light is spectrally narrow.

$$I_\nu(L) = I_\nu(0) \cdot \exp(+\alpha(\nu)\, L)$$

a sharply-peaked function of $\nu$

## Population inversion

We had for the small-signal absorption coefficient at frequency $\nu$:

$$\alpha(\nu) \equiv g(\nu) \left( \frac{g_2}{g_1} N_1 - N_2 \right) B_{21} \frac{nh\nu}{c}$$

If we can arrange for $N_2 - \frac{g_2}{g_1} N_1 > 0$

then there will be optical gain.

$N_2 - \frac{g_2}{g_1} N_1$ is known as the population-inversion **!**

We can define a small-signal gain coefficient

$$\kappa(\nu) \equiv -\alpha(\nu) = g(\nu) \left( N_2 - \frac{g_2}{g_1} N_1 \right) B_{21} \frac{nh\nu}{c} \quad !$$

A laser exploits this gain to provide a high-intensity beam of light.

Big assumption: the Einstein relations derived in conditions of thermal equilibrium also hold when there is population-inversion (negative effective temperature).

## Example

Calculate the ratio of the ground state population to that of the excited state at room temperature, when the transition between the energy levels (which can be taken to be non-degenerate) corresponds to a transition in the visible spectral region.

## Solution

Take the wavelength to be 550nm.

At thermal equilibrium

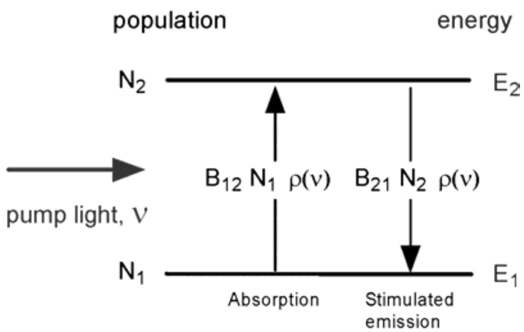
$$\frac{N_1}{N_2} = \frac{g_1}{g_2}\exp\left(\frac{h\nu}{k_B T}\right) = \frac{g_1}{g_2}\exp\left(\frac{hc}{k_B T\lambda}\right)$$

So $$\frac{N_1}{N_2} = \exp\left(\frac{6.63\times10^{-34}\times3.00\times10^{8}}{1.38\times10^{-23}\times300\times550\times10^{-9}}\right)$$

$$\approx 9\times10^{37}$$

## Achieving population inversion

Optical pumping alone on the transition on which gain is desired can never work!




The process reaches equilibrium when

$$N_2\,B_{21}\,\rho(\nu) = N_1\,B_{12}\,\rho(\nu)$$

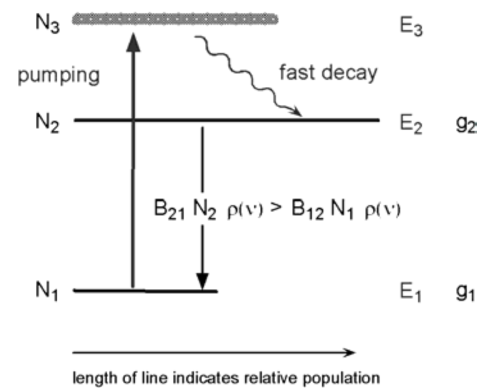
Since $g_1\,B_{21} = g_2\,B_{12}$

we find $N_2 - \frac{g_2}{g_1} N_1 = 0$

and so $k(\nu) = 0$

(in fact it is worse than this, as we neglected $A_{21}$)

- 3-Level system

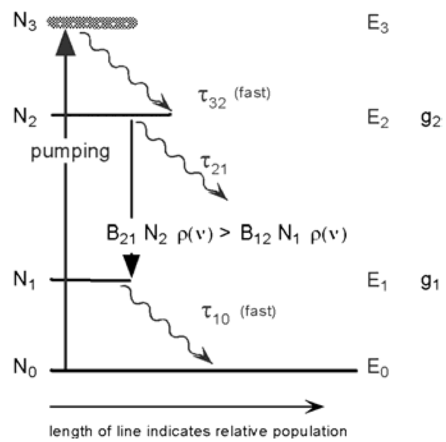



- Disadvantage:

Since the lower level of the laser transition in the atomic ground state, at least half the atoms have to be pumped into the excited state.

- E.g. Ruby $\lambda = 694\ nm$

• 4-Level system




A population inversion between states 1 and 2 can be maintained with only a small fraction of the atoms in excited states.

e.g. The neodymium-YAG laser has

$$\tau_{32} \approx 10^{-8}\,s$$
$$\tau_{21} \approx 0.5\,ms$$
$$\tau_{10} \approx 3\times 10^{-8}\,s$$

## General scheme for laser operation

Aim:

To produce a high-intensity, spectrally-pure light source

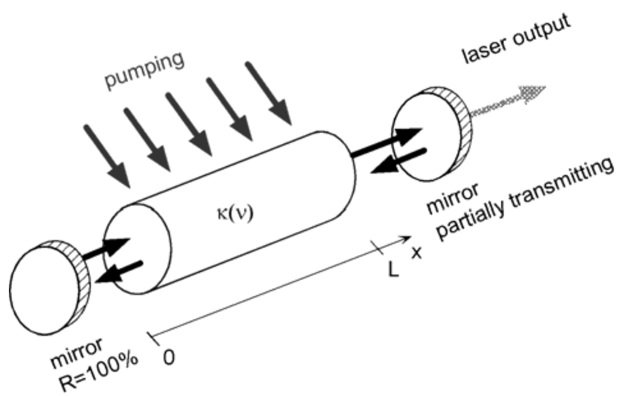
Problem:

Generally low gain per unit length

Consequence:

A single-pass amplifier suffers from
• small amplification,
• extraction of small fraction of pump energy

Solution:




Place the gain medium in an optical cavity

This allows for multiple-passes of the amplifying medium

(analogous to an amplifier with positive feedback)

Consequences of using a cavity:

- high intensity output
- efficient pump energy conversion
- low divergence, directed beam emission (analysis in terms of Gaussian beams)
- The laser frequency spectrum consists of a number of discrete lines associated with allowed modes of the cavity (possibility of single-frequency emission)

## Laser modes

• Longitudinal (axial) modes

The simplest case is the plane-plane cavity for which stable rays travel along the cavity axis.

The intra-cavity field must have a node at the mirrors.

Then $k\,L = \frac{2\pi n\nu}{c} L = m\pi$ for integer m

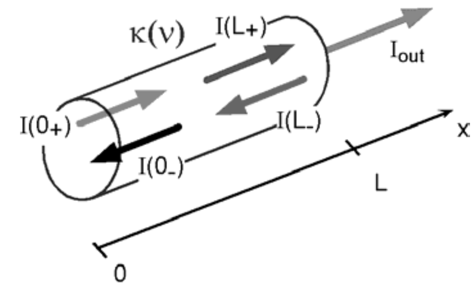
or $\nu = m \cdot \frac{c}{2nL}$

So the cavity-mode spacing $\Delta\nu_{cav} = \frac{c}{2nL}$

For intermediate values of m, a stable standing-wave cannot exist in the cavity

Laser action can only be obtained at the frequency of a cavity mode

## The round-trip gain




$$I(L_+) = I(0_+)\exp[(\kappa-\gamma)L]$$

Positive-travelling wave

After reflection off M1

$$I(L_-) = R_1\, I(0_+)\exp[(\kappa-\gamma)L]$$

After a complete round-trip

$$I(2L_+) = R_1\, R_2\, I(0_+)\exp[2(\kappa-\gamma)L]$$

## Threshold gain

Steady-state operation is possible for a round-trip gain equal to unity:

$$I(2L_+) = I(0_+)$$

$$= R_1\, R_2\, I(0_+) \exp[2(\kappa_{th} - \gamma)L]$$

This occurs at the threshold gain coefficient $\kappa_{th}$ where

$$\kappa_{th} = \gamma - \frac{1}{2L} \ln(R_1\, R_2) \qquad !$$

## Losses and the threshold gain

Sources of loss

- the useful output power of the laser;
- absorption and scattering at the cavity mirrors;
- loss of population from the energy levels involved in the laser transition;
- absorption and scattering at inhomogeneities in the gain medium;
- diffraction losses at the mirrors (and other intra-cavity elements)

The sum of these losses may be characterized by a loss coefficient, $\gamma$

Net gain coefficient is $\kappa - \gamma$

## Dipole emission patterns and scattering losses

Individual atoms emit radiation with a dipole angular distribution

**Q.** How can we reconcile this with the statement that for stimulated emission the emitted photon travels in the same direction as the incident photon?

$$\vec{k}_{emitted\ photon} = \vec{k}_{incident\ photon}$$

**A.** Atoms throughout the gain medium emit stimulated photons that are coherent

- The dipolar emission patterns of individual atoms interfere.
- Provided the emitting atoms are uniformly distributed in space, the intereference of dipole emission patterns gives a resultant amplitude distribution that reproduces that of the incident beam.
- Spatial inhomogeneity of gain medium gives rise to scattering losses.

## Population-inversion and the pumping threshold

We had, for the gain coefficient:

$$\kappa(\nu_s) = \frac{N^* B_{21}\, nh\nu_s\, g(\nu_s)}{c}$$

where $N^* \equiv \left(N_2 - \frac{g_2}{g_1} N_1\right)$ is the population inversion

Writing $g(\nu_s) \approx \frac{1}{\Delta\nu}$

and $B_{21} = \frac{c^3}{8\pi h\nu_s^3 n^3} A_{21} = \frac{c^3}{8\pi h\nu_s^3 n^3 \tau_{21}}$

gives $\kappa(\nu_s) = \frac{N^* c^2}{8\pi\, n^2 \nu_s^2\, \tau_{21}\, \Delta\nu}$

so, at threshold

$$N^* = N^*_{th} = \frac{8\pi\, \nu_s^2\, n^2\, \tau_{21}\, \kappa_{th}(\nu_s)\, \Delta\nu}{c^2} \quad !$$

Example

Find the Doppler width of the carbon-dioxide laser transition at wavelength, $\lambda$=10.6 $\mu$m, assuming the laser operates at 300K.

Hence find the population inversion required to give a small-signal gain coefficient of 1 $m^{-1}$ for a carbon-dioxide laser , for which the Einstein A-coefficient of the upper laser level is 200 $s^{-1}$

Find the pump power required to give the above value of the gain coefficient.

Solution

Using $\Delta\nu_{Doppler} = \frac{2}{\lambda_s}\sqrt{\frac{2k_B T}{m}}$

$$m_{CO_2} = 44\ a.u.$$

gives $\Delta\nu_{Doppler} = 6.2 \times 10^7\ Hz$

Then using $\kappa(\nu_s) = \frac{N^* B_{21}\, nh\,\nu_s\, g(\nu_s)}{c}$

with $B_{21} = \mathrm{A}_{21}\frac{c^3}{8\pi h n^3 \nu^3}$ and $g(\nu_s) \approx \frac{1}{\Delta\nu_{Doppler}}$

$$N^* = \frac{8\pi\, n^3 \kappa(\nu_s)\, \Delta\nu_{Doppler}}{A_{21}\ \lambda_s^2}$$

Giving, finally $N^* = 7.0\times10^{10}\ cm^{-3}$

## Gain saturation

The output power cannot increase indefinitely!

High optical power in the cavity ◆
high-rate of stimulated emission ◆
reduced upper laser level population ◆
reduced gain

Q. How far is the gain reduced?

A. To the threshold gain, $\kappa_{th}$ where
round-trip gain = round-trip loss

$$I(2L_+) = I(0_+) = R_1 R_2 I(0_+) \exp[2(\kappa_{th} - \gamma)L]$$

$$◆ \quad \kappa_{th} = \gamma - \frac{1}{2L} \ln(R_1 R_2) \quad !$$

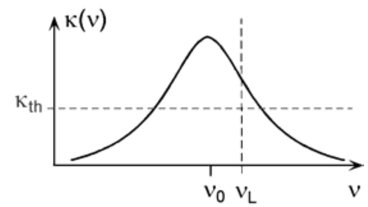
## Gain saturation in homogeneously-broadened transitions

Consider the case in which one cavity mode lies in the spectral region of the active medium gain curve.

All atoms contribute to all parts of the medium gain profile

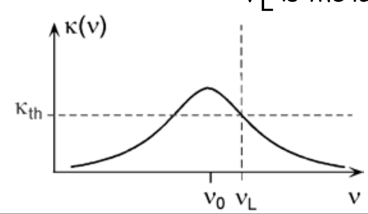
⇨ all atoms interact with the intra-cavity radiation

⇨ saturation decreases the entire gain profile




Unsaturated gain

$\nu_L$ is the laser operating frequency



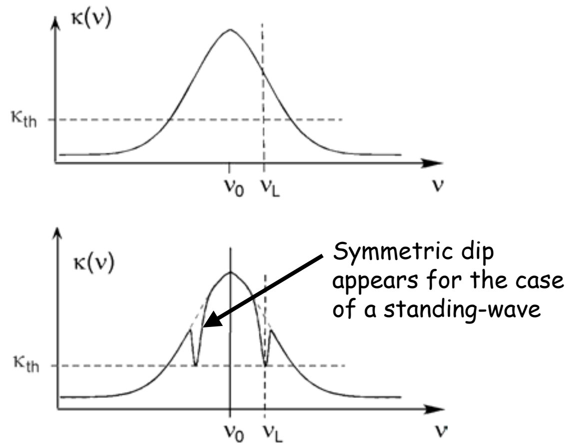

Saturated gain

Gain saturation in inhomogeneously-broadened transitions

Different classes of atoms contribute to different parts of the medium gain profile

⇨ only certain classes of atoms interact with the intra-cavity radiation

⇨ saturation locally decreases the gain to the threshold value




## Rate-equation analysis

We now have expressions for the threshold gain and the population inversion at threshold.

The aim now is to find the condition that must be satisfied by the pumping mechanism to achieve the threshold population inversion.

To do this, we shall use a rate-equation analysis

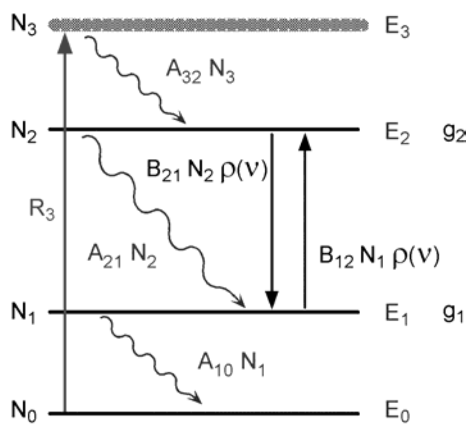
This approximate treatment neglects:

• propagation effects on the phase of the intra-cavity light,

• quantum-mechanical superpositions of atomic energy levels

• Rate-equation analysis of a four-level system

We consider the ideal 4-level system shown below




NB. In many laser systems, some direct pumping to the upper and lower laser levels is unavoidable; we neglect this here.

We have

$$\frac{dN_3}{dt} = R_3 - N_3\,A_{32}$$

$$\frac{dN_2}{dt} = N_3\,A_{32} - N_2\,A_{21} - N_2\,B_{21}\rho(\nu) + N_1\,B_{12}\rho(\nu) \qquad (1)$$

$$\frac{dN_1}{dt} = N_2\,B_{21}\rho(\nu) - N_1\,B_{12}\rho(\nu) + N_2\,A_{21} - N_1\,A_{10}$$

Solving for the steady-state and using the Einstein relation $g_2\,B_{21} = g_1\,B_{12}$ , these equations transform to

$$N_3\,A_{32} = R_3$$

$$N_2\left[A_{21} + B_{21}\rho(\nu)\right] = N_3\,A_{32} + N_1\frac{g_2}{g_1}B_{21}\rho(\nu)$$

$$N_2\left[A_{21} + B_{21}\rho(\nu)\right] = N_1\left[A_{10} + \frac{g_2}{g_1}B_{21}\rho(\nu)\right]$$

subtracting the last two yields $N_1 = \dfrac{R_3}{A_{10}}$

Then $N_2 = R_3 \cdot \dfrac{1 + \dfrac{g_2}{g_1}\dfrac{B_{21}}{A_{10}}\rho(\nu)}{A_{21} + B_{21}\rho(\nu)}$

So the steady-state population inversion

$$N^* \equiv N_2 - \frac{g_2}{g_1} N_1 = R_3 \cdot \frac{1 + \frac{g_2}{g_1}\frac{B_{21}}{A_{10}}\rho(\nu)}{A_{21} + B_{21}\rho(\nu)} - \frac{g_2}{g_1}\frac{R_3}{A_{10}}$$

$$N^* = R_3 \cdot \left( \frac{1 - \frac{g_2}{g_1}\frac{A_{21}}{A_{10}}}{A_{21} + B_{21}\rho(\nu)} \right)$$

Below and exactly at threshold $\rho(\nu)$ is small.

If it can be neglected, we can write

$$N^* = \frac{R_3}{A_{21}} \cdot \left( 1 - \frac{g_2}{g_1}\frac{A_{21}}{A_{10}} \right) \qquad !$$

N* increases linearly with the pump rate below threshold

To achieve threshold we require a pumping rate

$$R_3^{th} = N_{th}^* \cdot \frac{A_{21}}{1 - \frac{g_2}{g_1}\frac{A_{21}}{A_{10}}}$$

To achieve laser operation it is desirable that $\frac{A_{21}}{A_{10}} << 1$

which gives $R_3^{th} \approx N_{th}^* \cdot A_{21}$ !

Since $A_{21} = \frac{1}{\tau_{21}}$ we have, for the pumping power at threshold

$$P_{th} \approx N_{th}^* \cdot A_{21} \cdot (E_3 - E_0) \qquad !$$

We had $N_{th}^* = \dfrac{8\pi\, \nu_s^2\, n^2\, \tau_{21}\, \kappa_{th}(\nu_s)\ \Delta\nu}{c^2}$

The highest gain is produced at $\nu_s = \nu_0$ .
In this case

$$P_{th} = \frac{8\pi\,(E_3 - E_0)\ \nu_0^2\, n^2\, \kappa_{th}(\nu_0)\ \Delta\nu}{c^2} \qquad !$$

• Operation above threshold

At threshold, the optical gain induced by the pumping exactly balances the losses.

In the steady-state, the cavity round-trip gain must always equal the round-trip loss, or the intra-cavity energy density $\rho(\nu_L)$ would increase without limit.

Thus, in a steady-state situation, the population inversion never exceeds its value at threshold. So we have

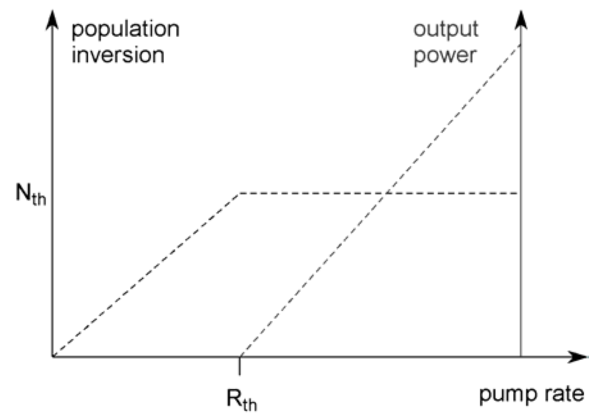
$$N_{th}^{*} = R_3^{th} \cdot \left( \frac{1 - \frac{g_2}{g_1}\frac{A_{21}}{A_{10}}}{A_{21}} \right) = R_3 \cdot \left( \frac{1 - \frac{g_2}{g_1}\frac{A_{21}}{A_{10}}}{A_{21} + B_{21}\rho(\nu)} \right)$$

◆ $$\rho(\nu) = \frac{A_{21}}{B_{21}} \cdot \left( \frac{R_3}{R_3^{th}} - 1 \right)$$

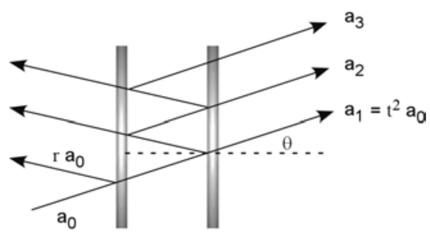
Since the output power, W, is proportional to $\rho(\nu)$

$$W = \mathrm{W}_0 \cdot \left( \frac{P}{P_{th}} - 1 \right) \qquad !$$

Graph of population inversion and laser output power vs. pump power.



• Laser cavities and Fabry-Perot etalons




$$a_1 = a_0\, t \exp ikL \cdot t$$
$$a_2 = a_0\, t \exp ikL \cdot r^2 \exp 2ikL \cdot t \qquad k \equiv \frac{4\pi}{\lambda}\cos\theta$$
$$a_3 = a_0\, t \exp ikL \cdot \left(r^2 \exp 2ikL\right)^2 \cdot t$$
$$\ldots$$

$$a_{out} = a_0\, t^2 \exp ikL \cdot \sum_{m=0}^{\infty} r^{2m} \exp 2mikL$$

$$a_{out} = a_0\, t^2 \exp ikL \cdot \frac{1}{1 - r^2 \exp 2ikL}$$

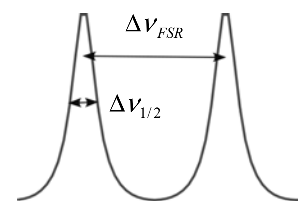
$$\left|\frac{a_{out}}{a_0}\right|^2 \propto \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2 kL}$$

$$T = \left|\frac{a_{out}}{a_0}\right|^2 \propto \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2 kL}$$ Airy function

• maxima: $\sin^2 kL = 0$

$$\Rightarrow k\,L = N\pi$$

$$\Rightarrow \nu = N \cdot \frac{c}{2nL}, \quad (\theta = 0)$$




The full-width at half-maximum of the peaks is given by

$$\frac{\Delta\nu_{1/2}}{\Delta\nu_{FSR}} = \frac{1}{F}$$

where the free-spectral range, $\Delta\nu_{FSR} = \frac{c}{2nL}$

and the finesse $F = 2\pi\,\frac{\sqrt{R}}{1-R}$

The operational linewidth of a laser is much less that the width of the Airy peaks of the corresponding bare Fabry-Perot etalon

because of the intra-cavity gain

$$R_{effective} \approx 1$$

hence $$F_{active} >> \frac{4R}{(1-R)^2}$$

It is easy to see that this should be the case by considering the cavity Q-factor

$$Q = \frac{resonant\ \ frequency}{linewidth}$$

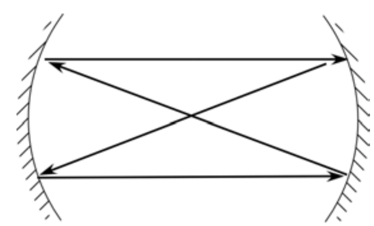
$$= \frac{2\pi \cdot energy\ stored}{energy\ dissipated\ \ per\ cycle}$$

which can be >>1 for a laser. This is an important distinction between a laser cavity, containing an active gain medium, and a bare Fabry-Perot cavity.

• Transverse modes

Plane-plane cavities are very lossy, so a laser cavity usually has at least one curved mirror.

In such a situation, rays whose propagation is not restricted to the axial direction can still form stable, closed paths and there is a spectrum of transverse modes.



These modes can be properly described as Gaussian beams, which we shall meet later.

Transverse modes are designated $TEM_{mn}$ (for Transverse Electromagnetic) where m (n) is the number of field nodes in the transverse x (y) direction.

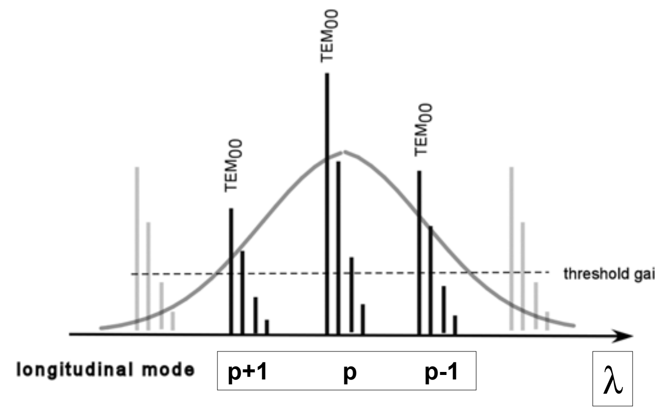
For the lowest-order transverse mode, $TEM_{00}$, the radial field distribution is Gaussian and all parts of the propagating wave-front are in phase (hence the designation `uniphase') - higher modes have phase reversals between anti-nodes


Example transverse-mode field distributions:




The signs refer to the relative phases of the field at the anti-nodes


• Complete cavity mode spectrum



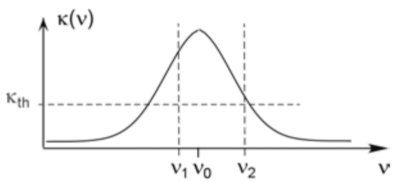

modes above threshold

• Higher-order modes often suffer greater losses (e.g. diffraction) than $TEM_{00}$ and are therefore less intense.

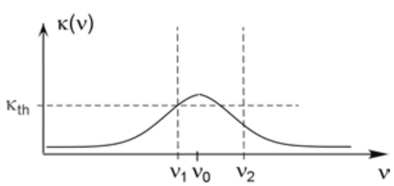
## The frequency spectrum of laser light

• Homogeneously-broadened case

If more than one cavity mode lies in the region for which the small-signal gain is above threshold,
gain saturation ensures that only the mode that is closest to line-centre oscillates




Unsaturated gain



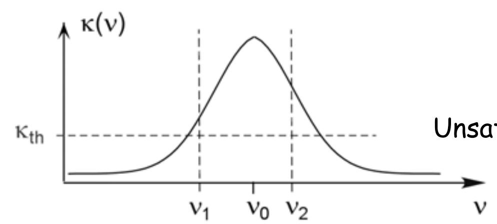

Laser operation

Single longitudinal-mode operation at frequency $\nu_1$

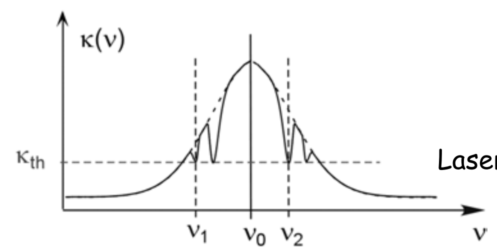
• Inhomogeneously-broadened case

Gain saturation locally decreases the gain to the
threshold value

⇨ all cavity modes for which the small-signal gain is above threshold can oscillate




Unsaturated gain




Laser operation

(We shall see later how operation on a single cavity mode can be enforced.)

## Single-mode operation

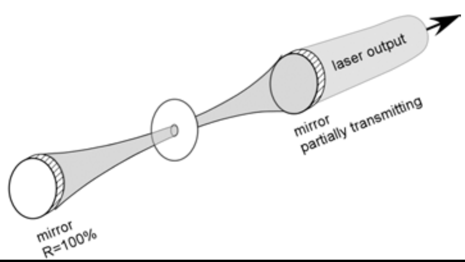
• In the steady-state, an ideal homogeneously broadened laser oscillates on the mode which has the greatest gain.

Inhomogeneously broadened systems need more help.

⇨ To select a particular mode, increase the losses for the undesired modes.

• Single transverse mode operation

• Restrict operation to $TEM_{00}$ transverse mode by
putting a pin-hole in the laser cavity:




• Single longitudinal-mode operation:

Use one (or more) intra-cavity etalons: