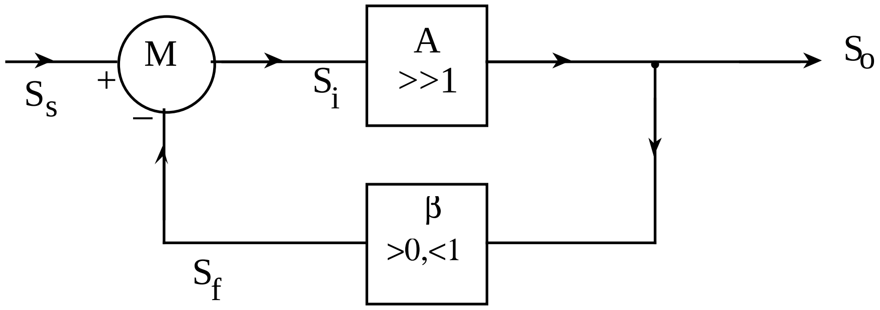
Week 9: Operational Amplifiers and Negative Feedback

9.1 Mathematical treatment of feedback

The general principle of a feedback circuit is that a fraction of the output of an amplifier is added or subtracted to the input. Addition gives positive feedback which leads to oscillation or other kinds of instability, subtraction gives negative feedback which leads to amplification with a reduced gain but with many compensating desirable other effects.

The general scheme for negative feedback can be represented as follows:



A is an amplifier with gain factor +*A*, and β is the feedback network which multiplies the output by the factor β (<1) before it is subtracted from the input to the circuit. “M” signifies the circuit element that subtracts the feedback from the input. If β (or *A*) is a negative number then the diagram will represent positive feedback.

The input, output, and feedback signals could be voltages or currents, so to make our treatment sufficiently general we signify these signals by the letter *S:*

S_s is the signal source – the input to the complete circuit,

S_i is the input to the amplifying element *A*,

S_o is the output of the amplifying element and of the complete circuit,

S_f is the feedback signal.

The equations relating these various signals are easily seen to be:

$$S_i = S_s - S_f$$
$$S_o = AS_i$$
$$S_f = \beta S_o.$$

Eliminating S_i and S_f we obtain

$$AS_i = AS_s - AS_f$$
$$S_o = AS_s - A\beta S_o$$
$$S_o = \left(\frac{A}{1+\beta A}\right)S_s.$$

Now if we have *A* very large such that $\beta A \gg 1$ we find

$$S_o = S_s / \beta,$$

so the overall gain of the circuit is simply $1/\beta$, independent of A. Since β is usually obtained by a simple potential divider circuit using resistors or a resistor and a capacitor, the gain of the circuit is fixed and constant with a well-defined dependence on the signal frequency – provided that the feedback condition $\beta A \gg 1$ remains true.

9.2 Feedback nomenclature

To distinguish the gain of the amplifying element, A, from the overall gain of the circuit, we call the former the *open loop gain* (sometimes written A_o) and the latter the *closed loop gain* (A_f).

9.3 Input and output impedance with negative feedback

As well as giving a stable well-defined gain, NFB affects considerably the input impedance and output impedance of the circuit, usually in a desirable fashion. How this happens depends on whether the feedback is voltage or current feedback.

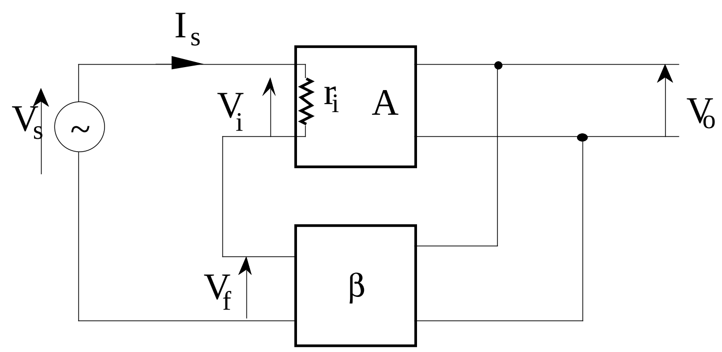
With voltage NFB a fraction of the output voltage is subtracted from the source voltage by placing them in series. This results in a large increase in input impedance and a large reduction in output impedance.

With current NFB, a fraction of the output current is subtracted from the source current by placing them in parallel. This results in a large decrease in input impedance and a large increase in output impedance.

There are other configurations of negative feedback; they are described later in section 9.5.

We will treat the general case for voltage NFB but rely on specific examples to illustrate current NFB.

INPUT IMPEDANCE



Let the amplifying element have an intrinsic input resistance r_i. Now the input current,

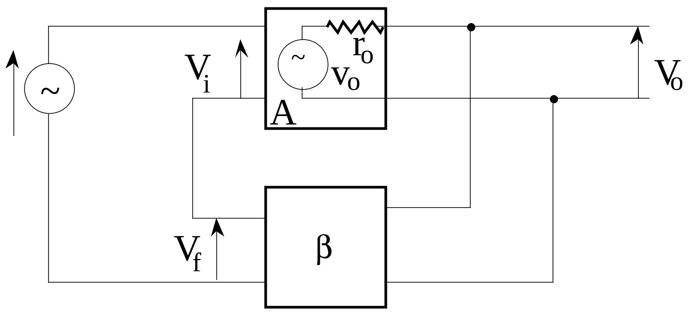
$$I_s = I_i = V_i/r_i = V_o/(Ar_i) \approx V_s/(\beta A r_i), \text{ [taking } V_o \approx V_s/\beta]$$

so the effective input resistance of the whole circuit is

$$r_{if} \equiv V_s/I_s = \beta A r_i \approx r_i A/A_f.$$

We therefore see that the input resistance is *increased* by a factor of (βA), which is $\gg 1$ in a normal NFB circuit.

OUTPUT IMPEDANCE
When we give the amplifying element an internal output resistance then we must distinguish the internal output voltage of the amplifier, v_o, from the output at its terminals, V_o.



Then (taking $V_S = 0$) the output current is

$$I_o = (v_o - V_o)/r_o = (AV_i - V_o)/r_o = -(AV_f+V_o)/r_o = -(A\beta V_o+V_o)/r_o$$
$$= -(A\beta+1)V_o/r_o\ .$$

The effective output resistance is therefore

$$r_{of} \equiv -V_o/I_o = r_o/(A\beta+1) \approx r_o/(A\beta) \approx r_o A_f/A.$$

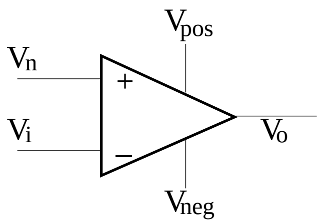
We therefore have a *reduction* in output resistance by a factor of $(A\beta)$, which is »1 in a normal NFB circuit.

[Note: the minus sign in the expressions for r_o and r_{of} can be understood by thinking of measuring them by applying a voltage to the output terminals and measuring the current flow: the current would flow *into* the amplifier. Alternatively, think of connecting a load resistance and measuring the output voltage and current as the load value is changed: a plot of V_o against I_o would have a negative slope.]

9.4 The operational amplifier

Circuits with negative or positive feedback are particularly simple to implement using an integrated circuit called an *operational amplifier* ("op amp" for short). It has two input terminals: *inverting* and *non-inverting*, and the two power supply connections are both required to be biassed away from earth potential: one positive and one negative. The output voltage is able to approach only within a few volts of either the positive or the negative supply voltage (2V in *Crocodile Clips*).

Here is the circuit symbol:



(Note: this is similar or identical to the symbol for an analogue comparator.)

An *ideal* op amp would obey the equation

$$V_o = A(V_n - V_i).$$

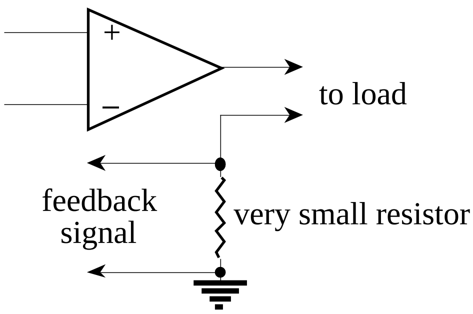
The intrinsic gain of the op amp, A, is very large: around 10^5. This means that for the output to be within, say, 0.1 V of earth (mid-way between V_{pos} and V_{neg}), the differential input voltage ($V_n - V_i$) would need to be less than 1µV, i.e. *minute*. In practice – even with the two input terminals connected together – there is an effective differential input voltage because of slight variations in the parameters of the transistors inside the integrated circuit. This "error" is called the *input offset voltage* and it is usually around 0.1 to 1 mV: much larger than the necessary input voltage estimated above.

Many op amps have two additional terminals for the connection of a potentiometer to allow the i.o.v. to be trimmed to very close to zero but it is rarely possible to get it down to 1µV. For this reason the op amp can only work in practice if it is part of a circuit containing positive or negative feedback. As we saw in the previous section, with negative feedback the exact value of the input doesn't matter. So long as it is small and A is large the circuit will come into balance.

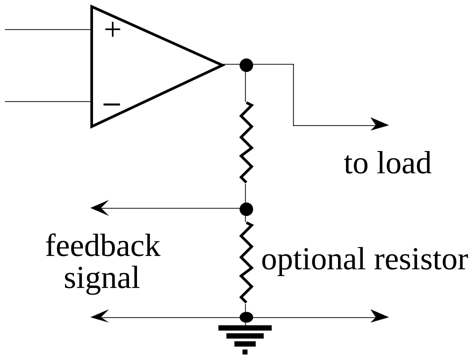
9.5 Negative feedback configurations using op amps

There are two ways of *deriving* the feedback signal: as a fraction of the output voltage or as a fraction of the output current. There are also two ways of *applying* the feedback signal to the input: in series or in parallel. All four permutations can be used.

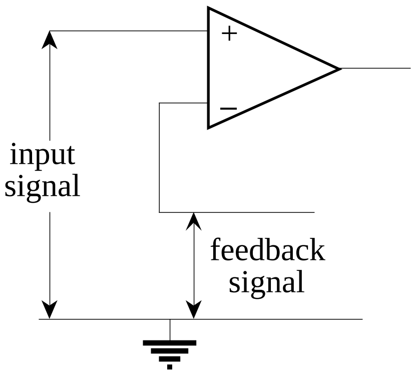
Here is how it's done using op amps:



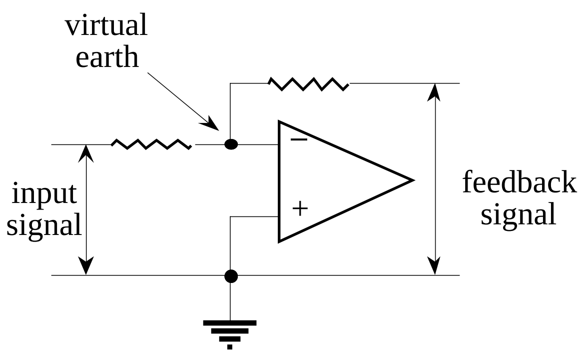
current-derived feedback



voltage-derived feedback



series-applied feedback



parallel-applied feedback

The resistor marked “optional” in the voltage-derived feedback circuit is not usually needed with parallel-applied feedback since the feedback current is determined by the other resistor alone.

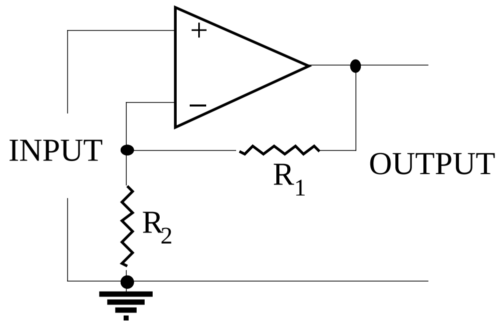
Note that series-applied feedback gives a non-inverting amplifier whereas parallel-applied feedback gives an inverting amplifier.

The input resistance of the parallel-applied circuit is just the value of the input resistor, which will not usually be extremely large, so for a very large input resistance we must use series-applied feedback.

Current-derived feedback is rarely used since most circuit designs are specified in terms of signal *voltages*. Current-derived feedback gives an amplifier whose output is a “constant current” signal for a given input, i.e. one having very large output impedance.

9.6 Non-inverting high-input resistance amplifier

Here we use voltage-derived series-applied feedback (“series-voltage” feedback or simply “voltage” feedback).



$\beta = R_2 / (R_1 + R_2)$, so the voltage gain is $A_f = (R_1 + R_2) / R_2 = (1 + R_1 / R_2)$.

In the special case where $R_1 = 0$, R_2 is irrelevant and can be omitted. We then have a unity-gain voltage amplifier.

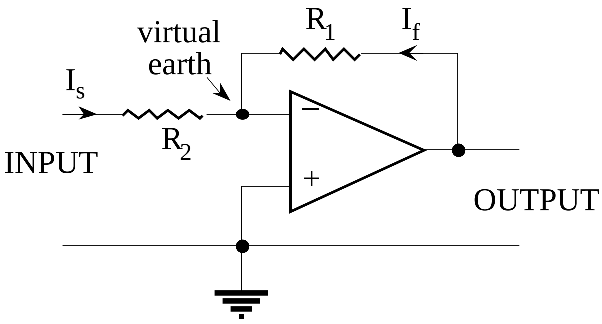
The operation of this circuit is easily visualised if we remember that for negative feedback with a large open-loop gain the input to the amplifying element will always be very small ($V_n – V_i = V_o / A \approx 0$). The potential at the inverting input to the op amp is therefore virtually equal to that at the non-inverting input, which in this case is just the input to the circuit. It is then obvious that the circuit gain is the inverse of the feedback factor.

Op amps have a differential input resistance (measured between their input terminals) of 100 kΩ or more, sometimes as high as 1 GΩ. If $A = 10^5$ then the circuit input resistance will be at least $10^5 \times 10^5 / A_f\ \Omega$: 10 MΩ if $A_f = 1000$, even with an op amp that does not have an especially large differential input resistance.

[Note that the *common-mode* input resistance – measured between joined-together input terminals and ground – is in *parallel* with the circuit input resistance so it is not affected by negative feedback and it is not possible to obtain a circuit input resistance greater than this value. Since it normally has a very high value in most cases its effect can be neglected.]

9.7 Parallel-voltage negative feedback: *the virtual earth*

The circuit of the inverting voltage amplifier is:



The standard NFB analysis, using a feedback factor β, would now have to be done in terms of feedback *current* and source *current*. However, we can avoid having to do this once we understand the operation of the *virtual earth*.

Here, the non-inverting input to the op amp is connected directly to earth. If the negative feedback is working properly then the inverting input must also be maintained at a potential virtually equal to earth by the feedback circuitry. Hence this point in the circuit is referred to as a *virtual earth*. It differs from the real earth, however, in that approximately zero current can flow into it. This is because the potential is virtually zero *and* the input resistance of the op amp is quite high.

The input current to the circuit, therefore, must be precisely minus the feedback current:

$$I_f = -I_s.$$

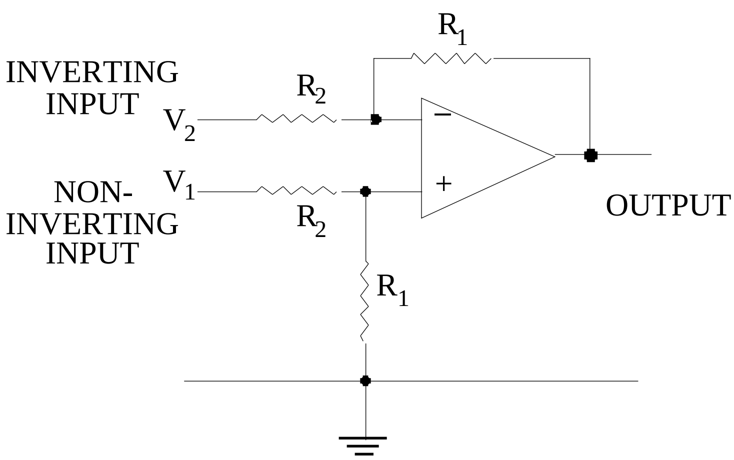
This is the design principle for this kind of negative feedback circuit.

Applying Ohm's law to R_1 and R_2:

$$I_f = V_o / R_1 \qquad I_s = V_s / R_2$$
$$\text{so } A_f = V_o / V_s = -R_1 / R_2.$$

9.8 The voltage difference amplifier

We can combine the non-inverting series-voltage feedback amplifier with the inverting parallel-voltage feedback amplifier to make a single circuit which acts as a difference amplifier of well-defined closed-loop gain.



If we apply an input only at V_2, then $V_1 = 0$ (i.e. earth), and the circuit is the same as the previous parallel-voltage feedback inverting amplifier with the addition of R_1 and R_2 in parallel between earth and the non-inverting input to the op amp. This makes no difference since there is a negligible current flowing into the op amp input. The closed loop-gain is therefore as before:

$$A_{f2} = V_o / V_2 = -R_1 / R_2.$$

If we apply an input only at V_1, then $V_2 = 0$ (i.e. earth),, and the circuit is the same as the previous series-voltage feedback non-inverting amplifier with the addition of a potential divider before the connection to the op amp. The closed loop-gain is therefore:

$$A_{f1} = V_o / V_1 = \{ (R_1 + R_2) / R_2 \} \times \{ R_1 / (R_1 + R_2) \} = R_1 / R_2.$$

Now if we apply signals to both inputs, because the equations of the circuit are linear, we can combine the two parts to obtain:

$$V_o = A_{f1} V_1 + A_{f2} V_2 = (V_1 - V_2) R_1 / R_2.$$

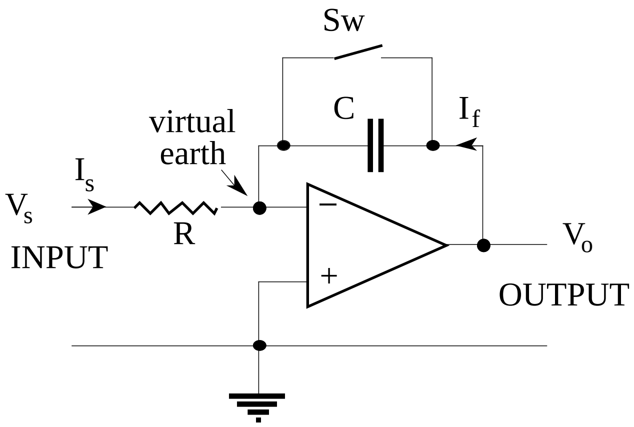
9.9 Other parallel-voltage negative feedback circuits

Because the potential differences across the two external components in the parallel-voltage NFB circuit are directly equal to the input and the output voltages, this circuit can be simply adapted to perform a wide range of desirable functions by replacing one or both of these resistors by other components, such as capacitors, diodes, or various series / parallel combinations of these.

The magnitudes of the currents flowing in the two components are always equal and the voltage across one is the input voltage and the voltage across the other is the output voltage.

9.10 The inverting integrator

Here, the feedback component is a capacitor.



$$I_s = V_s / R = -I_f = -C \frac{\mathrm{d}V_o}{\mathrm{d}t}$$

$$\therefore \quad \mathrm{d}V_o = -\frac{1}{RC} V_s \,\mathrm{d}t$$

$$V_o = -\frac{1}{RC} \int V_s \,\mathrm{d}t.$$

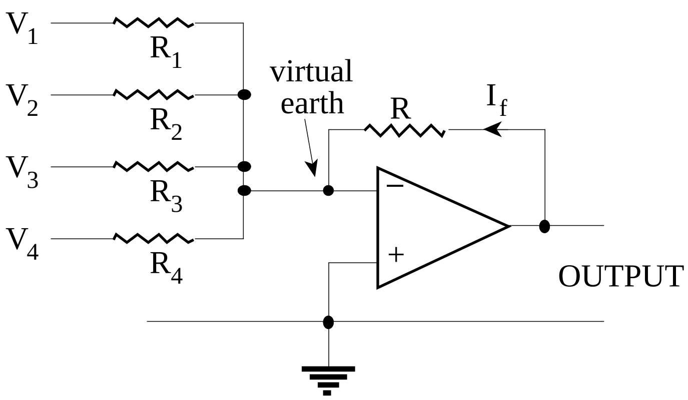
The output voltage is therefore the time-wise integral of the input signal. The time t=0 from which the integration begins – and at which the output voltage must be zero – is determined by holding the switch Sw closed until that instant. This switch can be a field-effect transistor (not available in *Crocodile Clips*), or for a slow circuit, a relay (electromagnetic switch).

9.11 The inverting differentiator

Interchanging R and C in the above circuit produces a differentiator (omitting the switch!). However, in practice, extra components are needed to prevent the output being swamped by high frequency noise.

This arises because at high frequency the impedance of the capacitor approaches zero, so the closed-loop gain approaches infinity. A small resistor placed in series with the capacitor and / or a small capacitor in parallel with the feedback resistor cures this.

9.12 The inverting analogue sum / mixer

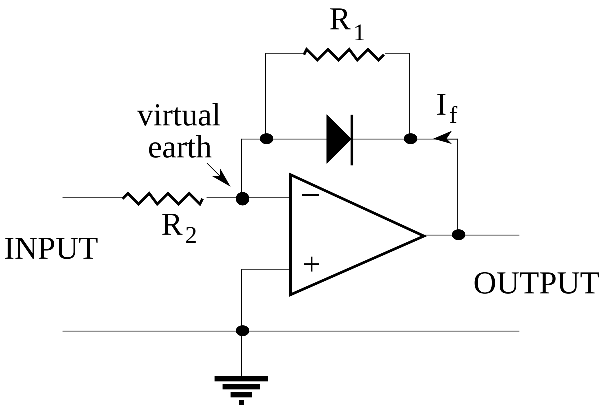


$$\frac{V_o}{R} = I_f = -\sum_i \frac{V_i}{R_i}$$

$$\therefore \quad V_o = -R \sum_i \frac{V_i}{R_i}.$$

When all $R_i = R$ then the output voltage is minus the sum of the input voltages.

9.13 The inverting rectifier



When the input is negative the output is positive and the diode is reverse biassed. The closed-loop gain is therefore $–R_1 / R_2$.

When the input is positive the output goes negative to about –0.6 V because the diode is then forward biassed.

The diode current is approximately V_s / R_2 (ignoring the effect of R_1) so the diode slope resistance is $0.04 R_2 / V_s$, hence the closed-loop signal gain is $0.04 / V_s$, so the output *signal* voltage is of the order of 40mV.

Using the exact forward characteristic of the diode we can see that for small negative output voltages this circuit works as an inverting logarithmic amplifier. Interchanging the resistor and the diode produces an inverting anti-log amplifier (exponentiator). Such circuits are the basis of analogue multipliers.

Exercises:
Construct using Crocodile Clips and using the real op amp board one or more of the circuits covered in this section and compare their behaviour.