ASTM-052 Extragalactic Astrophysics

EXERCISES: SET NUMBER 1

The questions are inhomogeneous in style and length and are not necessarily representative of examination questions. To see the *style*, but not the content, of the latter, see last year's paper. I suggest that you do not need do all the questions but should choose those on the examinable topics you are least happy with.

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1. Explain why Kapteyn's model of the Galaxy was (a) so much smaller and (b) so much thicker, relative to its diameter, than current models.

   Hubble's constant is expressed as

   $$H_o = h \times 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \tag{1.1}$$

   Show, by direct calculation, that

   $$\frac{1}{H_o} \approx \frac{10^{10}}{h} y \tag{1.2}$$

   Hubble's value of h was about 5. What was the problem with this value?

2. Sketch Hubble's scheme for the classification of galaxies.

   The ellipticity of a flat disc, viewed at an angle θ to its normal, is given by

   $$e = 1 - \cos\theta \tag{2.1}$$

   Show that, if all elliptical galaxies were flat discs inclined at various angles to our line of sight, then they would have to conspire to be so inclined by no more than about 75°.

3. Define what is meant by colour in astronomy.

   Sketch, on a (U-B) *versus* (B-V) colour-colour plot, the relative positions occupied by elliptical and the various classes of spiral galaxies.

   Interpret these positions in terms of the stellar populations of the galaxies.

4. Define what is meant by the surface brightness $I(\theta)$ of a galaxy.

   The surface brightness of elliptical galaxies, and of the nuclear regions of many spiral galaxies, is well described by the de Vaucouleurs profile:

   $$I(\theta) = I(0)\exp\left[-\left(\frac{\theta}{\theta_o}\right)^{1/4}\right]. \tag{4.1}$$

   Sketch this function.

   Show that the total flux-density F emitted by a galaxy with surface brightness $I(\theta)$ is given by

   $$F = \int_0^\infty I(\theta) 2\pi\theta \, d\theta . \tag{4.2}$$

   Use equation (4.2) to show that, for a de Vaucouleurs profile, F is given by

   $$F = 8!\pi\theta_o^2 I(0) \tag{4.3}$$

   [You may use without proof the following mathematical relationship:

   $$\int_0^\infty x^n e^{-x} dx = n! \,] \tag{4.4}$$

   Consider a fictitious galaxy with *uniform* surface brightness:

   $$I(\theta) = \begin{cases} I_o & \theta \le \theta_{max} \\ 0 & \theta > \theta_{max} \end{cases} \tag{4.5}$$

   Suppose that this fictitious galaxy has the same *central* brightness as the galaxy described by equation (4.3). Show that, if it is to have the same total flux, we must have

   $$\theta_{max} \approx 200\,\theta_o . \tag{4.6}$$

   Comment on this result.

   The function $J_E(R)$ of a spherically symmetric elliptical galaxy describes the way the volume emissivity changes with distance R from the centre of the galaxy. Explain **briefly** and **qualitatively**, with the help of a sketch if necessary, why $J_E(R)$ falls off *more* *rapidly* with R than does the surface brightness $I_E(\theta)$ with θ, R and θ being related by

$$\theta = \frac{R}{d}\,, \tag{4.7}$$

where d is the distance of the galaxy. Why does the same not apply to the discs of spiral galaxies?

5. Use the method of dimensions to show that the mass of a galaxy of characteristic size R is given by

$$M \sim \frac{Rv^2}{G}\,, \tag{5.1}$$

where v is a characteristic velocity of material in the galaxy and G the gravitational constant.

6. A stellar absorption line has a rest wavelength λ_o of 500 nm and an intrinsic width $\Delta\lambda$ of 0.05 nm. The *observed* width of this line in the spectrum of a large E0 galaxy is 0.503 nm. Deduce that the mean-squared line-of-sight velocity of the stars in the galaxy is about 300 km s^{-1}. [NB The natural line-width and the Doppler line-width add in quadrature.]

Estimate the mass (in solar masses) of the galaxy, given that its radius is about 50 kpc.

7. Describe briefly, with the aid of a diagram, how long-slit spectroscopy can be used to measure the rotation curve of a galaxy.

Assuming that the mass in a spiral galaxy is distributed with spherical symmetry about its centre, show that the mass contained within radius r of the centre is given by

$$M(r) = \frac{r\Theta^2(r)}{G}\,, \tag{7.1}$$

where $\Theta(r)$ is the circular velocity of material in the disc of the galaxy.

The inner part of the rotation curves of spiral galaxies increases approximately linearly out to a radius r_o:

$$\Theta(r) \approx \Theta_o \times \frac{r}{r_o}; \quad r \le r_o\,, \tag{7.2}$$

where Θ_o is the approximately constant circular velocity in the outer regions. Deduce that the distribution of mass with radius in these regions is given by

$$M(r) \approx \left(\frac{\Theta_o^2}{Gr_o^2}\right) r^3\,. \tag{7.3}$$

How must the density $\rho(r)$ vary in this region?

Distances of galaxies are obtained by measuring their redshifts z, calculating their recession velocities v_H and using Hubble's law to get the distance d:

$$d = \frac{v_H}{H_o} = z\frac{c}{H_o} \tag{7.4}$$

Show that the luminosity L of a galaxy is given, in terms of its *measured* flux-density F, by

$$L = 4\pi\left(z\frac{c}{H_o}\right)^2 F\,. \tag{7.5}$$

Masses of galaxies are derived from an expression of the form

$$M = k\frac{Rv^2}{G}\,, \tag{7.6}$$

R being some characteristic length-scale for the galaxy, v being some measured characteristic velocity of the material in it, and k being a constant. Deduce that

$$M = \left(z\frac{c}{H_o}\right)\frac{v^2}{G}\theta\,, \tag{7.7}$$

where θ is the *measured* angular size corresponding to the linear size R.

Hence show that the *measured* mass-luminosity ratio M/L of galaxies is directly proportional to the the Hubble constant:

$$\frac{M}{L} \propto H_o \tag{7.8}$$

What are the ranges of values, in solar units, of the mass-luminosity ratios of (a) elliptical and (b) spiral galaxies?

8. Apply the Euler equation to the density ρ of the fluid and use the equation of continuity to show that

$$\frac{D\rho}{Dt} = -\rho\nabla.\mathbf{u}\,. \tag{8.1}$$

Deduce that, for an *incompressible* fluid,

$$\nabla.\mathbf{u} = 0\,. \tag{8.2}$$

9. By considering the pressure forces acting on opposite sides of a small cube of fluid, show that the force $\mathbf{F}_p$ per unit mass of the fluid, arising from a pressure gradient ∇p, is given by

$$\mathbf{F}_p = -\frac{1}{\rho}\nabla p\,. \tag{9.1}$$

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