# PHYS2B22 Quantum Physics
# Evening course lecture notes. Set 1.
Sam Morgan 2005

## QUANTUM THEORY PHYS2B22 EVENING CLASS 2005

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**Website**
- http://www.tampa.phys.ucl.ac.uk/~sam/2B22.html
- Contains: Lecture notes, problem sets and past exam papers

**Timetable:** 11 sets of 3 hour lectures (with break!)
Mondays 6-9pm, Room A1,
Jan 10th to March 21st inclusive

**Assessment:** 90% on summer exam
10% on best 3 of 4 problem sheets
NB rules on exam withdrawals (Student Handbook p21)
NB 15% rule on coursework (Student Handbook p14-15)

## TEXTBOOKS

Main texts

**Alastair Rae** Quantum Mechanics (IoP) (£12 -- closest to course)
**Brehm and Mullin** Introduction to the structure of matter (Wiley)
(£26 -- general purpose book)
*Both available at a discount via the department*

Also useful

**Bransden and Joachain** Quantum Mechanics (Prentice Hall)
*(£29 -- also useful for more advanced courses)*
**R. Feynman** Lectures on Physics III (Addison-Wesley)
*(first 3 chapters give an excellent introduction to the main concepts)*

# SYLLABUS

***1. The failure of classical mechanics***

Photoelectric effect, Einstein's equation, electron diffraction and de Broglie relation. Compton scattering. Wave-particle duality, Uncertainty principle (Bohr microscope).

***2. Steps towards wave mechanics***

Time-dependent and time-independent Schrödinger equations. The wave function and its interpretation.

***3. One-dimensional time-independent problems***

Infinite square well potential. Finite square well. Probability flux and the potential barrier and step. Reflection and transmission. Tunnelling and examples in physics and astronomy. Wavepackets. The simple harmonic oscillator.

***4. The formal basis of quantum mechanics***

The postulates of quantum mechanics – operators, observables, eigenvalues and eigenfunctions. Hermitian operators and the Expansion Postulate.

***5. Angular momentum in quantum mechanics***

Operators, eigenvalues and eigenfunctions of $\hat{L}_z$ and $\hat{\underline{L}}^2$.

# SYLLABUS (cont)

***6. The hydrogen atom***

Separation of space and time parts of the 3D Schrödinger equation for a central field. The radial Schrödinger equation and its solution by series method. Degeneracy and spectroscopic notation.

***7. Electron spin and total angular momentum***

Magnetic moment of electron due to orbital motion. The Stern-Gerlach experiment. Electron spin and complete set of quantum numbers for the hydrogen atom. Rules for addition of angular momentum quantum numbers. Total spin and orbital angular momentum quantum numbers S, L, J. Construct J from S and L.

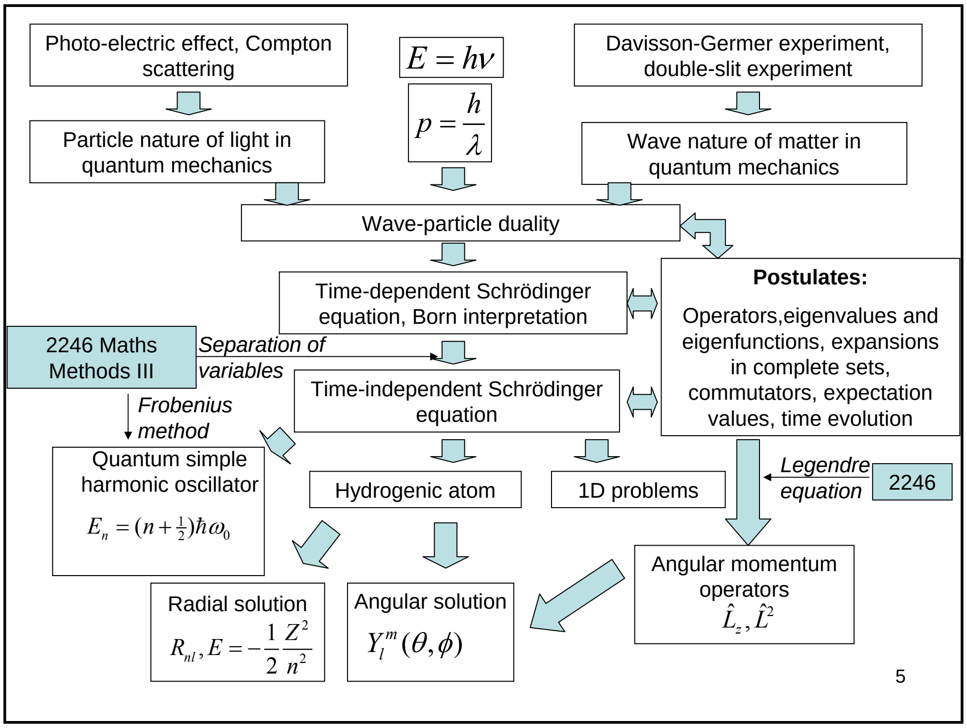

Photo-electric effect, Compton scattering

Particle nature of light in quantum mechanics

$E = h\nu$

$p = \frac{h}{\lambda}$

Davisson-Germer experiment, double-slit experiment

Wave nature of matter in quantum mechanics

Wave-particle duality

Time-dependent Schrödinger equation, Born interpretation

**Postulates:**

Operators, eigenvalues and eigenfunctions, expansions in complete sets, commutators, expectation values, time evolution

2246 Maths Methods III

*Separation of variables*

Time-independent Schrödinger equation

*Frobenius method*

Quantum simple harmonic oscillator

$E_n = (n+\tfrac{1}{2})\hbar\omega_0$

Hydrogenic atom

1D problems

*Legendre equation*

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Angular momentum operators

$\hat{L}_z, \hat{L}^2$

Radial solution

$R_{nl}, E = -\frac{1}{2}\frac{Z^2}{n^2}$

Angular solution

$Y_l^m(\theta,\phi)$

# WAVE PARTICLE DUALITY

Evidence for wave-particle duality

- Photoelectric effect
- Compton effect

- Electron diffraction
- Interference of matter-waves

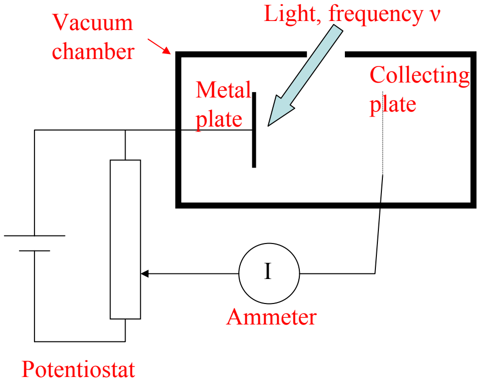
Consequence: Heisenberg uncertainty principle

## PHOTOELECTRIC EFFECT

**Hertz**

**J.J. Thomson**

When UV light is shone on a metal plate in a vacuum, it emits charged particles (Hertz 1887), which were later shown to be electrons by J.J. Thomson (1899).



**Classical expectations**

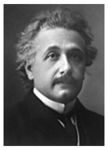
Electric field E of light exerts force **F**=-*e***E** on electrons. As intensity of light increases, force increases, so KE of ejected electrons should increase.

Electrons should be emitted whatever the frequency ν of the light, so long as **E** is sufficiently large

For very low intensities, expect a time lag between light exposure and emission, while electrons absorb enough energy to escape from material

## PHOTOELECTRIC EFFECT (cont)

**Einstein**



**Millikan**

**Actual results:**

Maximum KE of ejected electrons is independent of intensity, but dependent on ν

For $\nu<\nu_0$ (i.e. for frequencies below a cut-off frequency) no electrons are emitted

There is no time lag. However, rate of ejection of electrons depends on light intensity.

**Einstein's interpretation (1905):**

Light comes in packets of energy (*photons*)

$$E = h\nu$$

An electron absorbs a single photon to leave the material

The maximum KE of an emitted electron is then

$$K_{\max} = h\nu - W$$

*Planck constant*: universal constant of nature

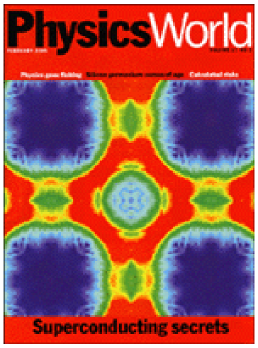
$h = 6.63\times10^{-34}\,\mathrm{Js}$

*Work function*: minimum energy needed for electron to escape from metal (depends on material, but usually 2-5eV)

Verified in detail through subsequent experiments by Millikan

## Photoemission experiments today

Modern successor to original photoelectric effect experiments is *ARPES (**A**ngle-**R**esolved **P**hoto**e**mission **S**pectroscopy)*



February 2000

Emitted electrons give information on distribution of electrons within a material as a function of energy *and* momentum

## SUMMARY OF PHOTON PROPERTIES

Relation between particle and wave properties of light

Energy and frequency $E = h\nu$

Also have relation between momentum and wavelength

Relativistic formula relating energy and momentum $\boxed{E^2 = p^2c^2 + m^2c^4}$

For light $E = pc$ and $c = \lambda\nu$

$$\boxed{p = \frac{h}{\lambda} = \frac{h\nu}{c}}$$

Also commonly write these as

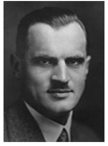
wavevector

$$E = \hbar\omega \qquad p = \hbar k \qquad \omega = 2\pi\nu \qquad k = \frac{2\pi}{\lambda} \qquad \hbar = \frac{h}{2\pi}$$

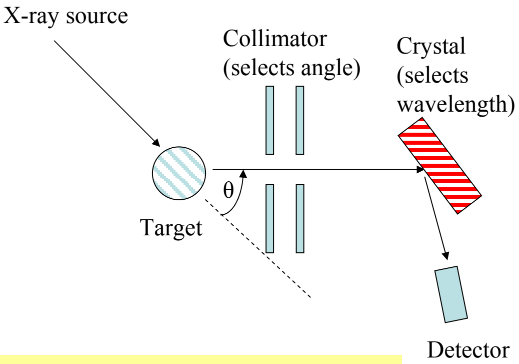
angular frequency hbar

# COMPTON SCATTERING

**Compton**



Compton (1923) measured intensity of scattered X-rays from solid target, as function of wavelength for different angles. He won the 1927 Nobel prize.



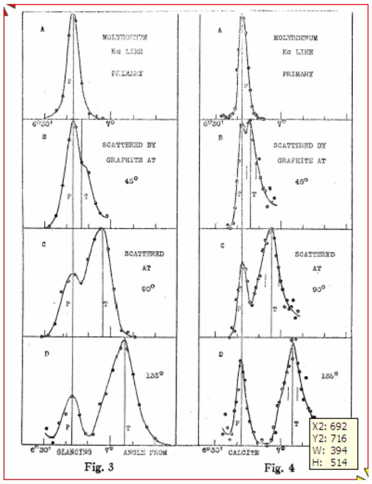


Fig. 3

Fig. 4

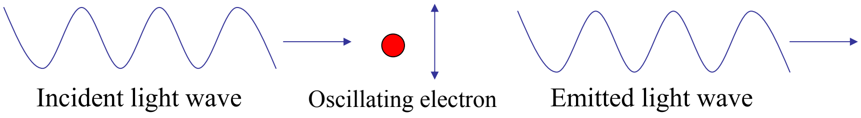
A.H. Compton, *Phys. Rev.* **22** 409 (1923)

**Result:** peak in scattered radiation shifts to longer wavelength than source. Amount depends on $\theta$ (but not on the target material).

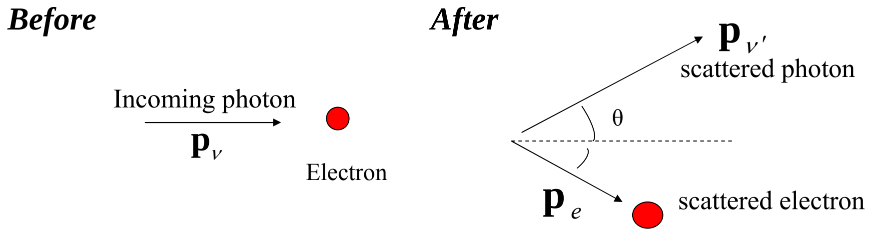
# COMPTON SCATTERING (cont)

**Classical picture:** oscillating electromagnetic field causes oscillations in positions of charged particles, which re-radiate in all directions at *same frequency and wavelength* as incident radiation.

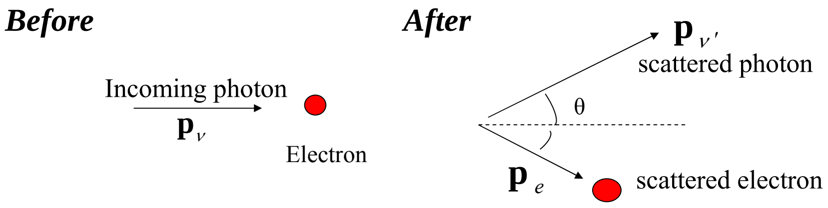
**Change in wavelength of scattered light is completely unexpected classically**



**Compton's explanation:** "billiard ball" collisions between particles of light (X-ray photons) and electrons in the material



## COMPTON SCATTERING (cont)



Conservation of energy

$$h\nu + m_e c^2 = h\nu' + \left(p_e^2 c^2 + m_e^2 c^4\right)^{1/2}$$

Conservation of momentum

$$\mathbf{p}_\nu = \frac{h}{\lambda}\hat{\mathbf{i}} = \mathbf{p}_{\nu'} + \mathbf{p}_e$$

From this Compton derived the change in wavelength

$$\boxed{\begin{aligned}\lambda' - \lambda &= \frac{h}{m_e c}\left(1-\cos\theta\right)\\ &= \lambda_c\left(1-\cos\theta\right) \geq 0\end{aligned}}$$

$$\lambda_c = \text{ Compton wavelength } = \frac{h}{m_e c} = 2.4\times10^{-12}\,\text{m}$$

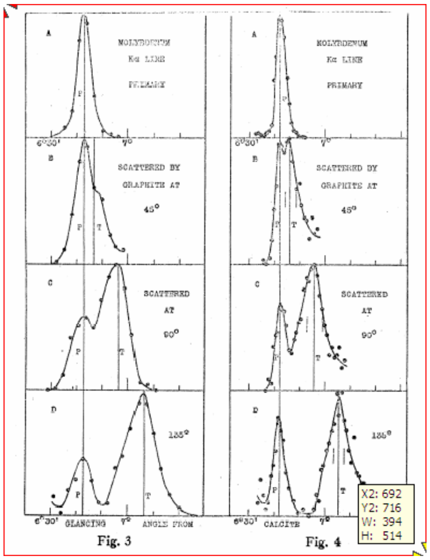
## COMPTON SCATTERING (cont)

Note that, at all angles
there is also an unshifted peak.

This comes from a collision between the X-ray photon and the nucleus of the atom

$$\boxed{\lambda' - \lambda = \frac{h}{m_N c}\left(1-\cos\theta\right) \sim 0}$$

since $m_N \gg m_e$



# WAVE-PARTICLE DUALITY OF LIGHT

In 1924 Einstein wrote:- “ There are therefore now two theories of light, both indispensable, and … without any logical connection.”

Evidence for wave-nature of light

- Diffraction and interference

Evidence for particle-nature of light

- Photoelectric effect
- Compton effect

- Light exhibits diffraction and interference phenomena that are *only* explicable in terms of wave properties
- Light is always detected as packets (photons); if we look, we never observe half a photon
- Number of photons proportional to energy density (i.e. to square of electromagnetic field strength)

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***De Broglie***

# MATTER WAVES

We have seen that light comes in discrete units (photons) with particle properties (energy and momentum) that are related to the wave-like properties of frequency and wavelength.

In 1923 Prince Louis de Broglie postulated that ordinary matter can have wave-like properties, with the wavelength $\lambda$ related to momentum $p$ in the same way as for light

de Broglie relation

$$\boxed{\lambda = \frac{h}{p}}$$

de Broglie wavelength

Planck’s constant

$h = 6.63 \times 10^{-34}\,\text{Js}$

NB wavelength depends on momentum, not on the physical size of the particle

**Prediction:** We should see diffraction and interference of matter waves

## Estimate some de Broglie wavelengths

- Wavelength of electron with 50eV kinetic energy

$$K=\frac{p^2}{2m_e}=\frac{h^2}{2m_e\lambda^2}\Rightarrow\lambda=\frac{h}{\sqrt{2m_eK}}=1.7\times10^{-10}\,\mathrm{m}$$

- Wavelength of Nitrogen molecule at room temperature

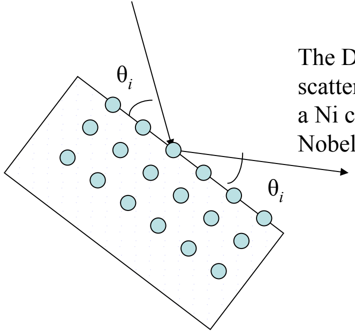
$$K=\frac{3kT}{2},\quad \mathrm{Mass}=28\mathrm{m_u}$$

$$\lambda=\frac{h}{\sqrt{3MkT}}=2.8\times10^{-11}\,\mathrm{m}$$

- Wavelength of Rubidium(87) atom at 50nK

$$\lambda=\frac{h}{\sqrt{3MkT}}=1.2\times10^{-6}\,\mathrm{m}$$

## ELECTRON DIFFRACTION
## The Davisson-Germer experiment (1927)

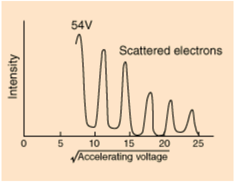


The Davisson-Germer experiment: scattering a beam of electrons from a Ni crystal. Davisson got the 1937 Nobel prize.

**Davisson**

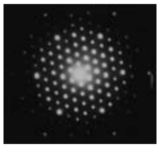
**G.P. Thomson**

At fixed *angle*, find sharp peaks in intensity as a function of electron energy



Davisson, C. J., "Are Electrons Waves?," Franklin Institute Journal **205**, 597 (1928)

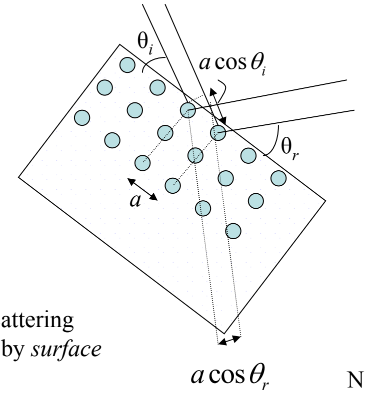
At fixed accelerating voltage (fixed electron energy) find a pattern of sharp reflected beams from the crystal



G.P. Thomson performed similar interference experiments with thin-film samples

## ELECTRON DIFFRACTION (cont)

**Interpretation:** similar to Bragg scattering of X-rays from crystals



**Path difference:**

$$a(\cos\theta_r - \cos\theta_i)$$

**Constructive interference when**

$$a(\cos\theta_r - \cos\theta_i) = n\lambda$$

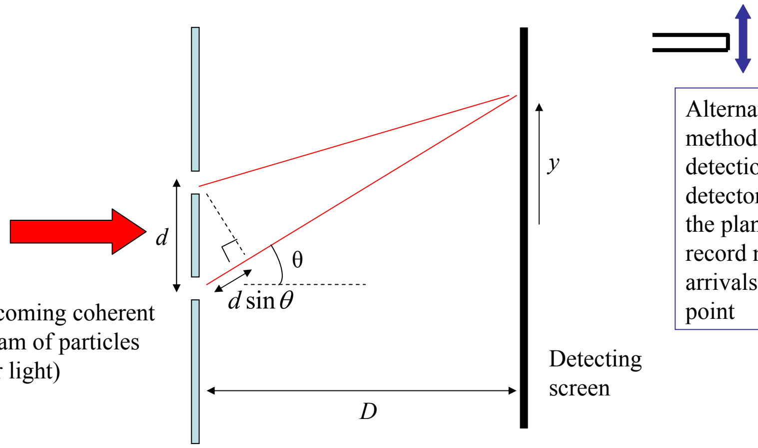
Electron scattering dominated by *surface* layers

Note $\theta_i$ and $\theta_r$ not necessarily equal

Note difference from usual “Bragg’s Law” geometry: the identical scattering planes are oriented *perpendicular* to the surface

## THE DOUBLE-SLIT EXPERIMENT

Originally performed by Young (1801) to demonstrate the wave-nature of light. Has now been done with electrons, neutrons, He atoms among others.



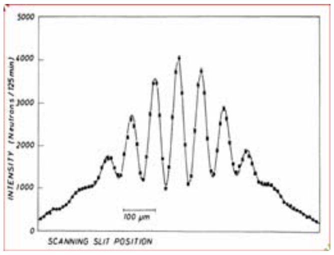
Incoming coherent beam of particles (or light)

Detecting screen

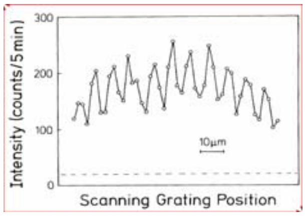
Alternative method of detection: scan a detector across the plane and record number of arrivals at each point

For particles we expect two peaks, for waves an interference pattern

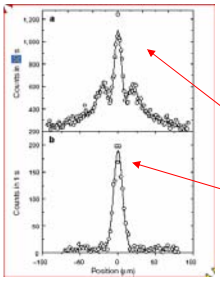
## EXPERIMENTAL RESULTS



Neutrons, A Zeilinger *et al.* 1988 *Reviews of Modern Physics* **60** 1067-1073



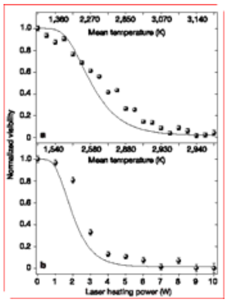
He atoms: O Carnal and J Mlynek 1991 *Physical Review Letters* **66** 2689-2692



$C_{60}$ molecules: M Arndt *et al.* 1999 *Nature* **401** 680-682

With multiple-slit grating

Without grating



Fringe visibility decreases as molecules are heated. L. Hackermüller *et al.* 2004 *Nature* **427** 711-714

Interference patterns can not be explained classically - clear demonstration of matter waves

## DOUBLE-SLIT EXPERIMENT WITH HELIUM ATOMS

(Carnal & Mlynek, 1991,Phys.Rev.Lett.,66,p2689)

**Path difference:** $d\sin\theta$

**Constructive interference:** $d\sin\theta = n\lambda$

**Separation between maxima:** $\Delta y = \dfrac{\lambda D}{d}$
*(proof following)*

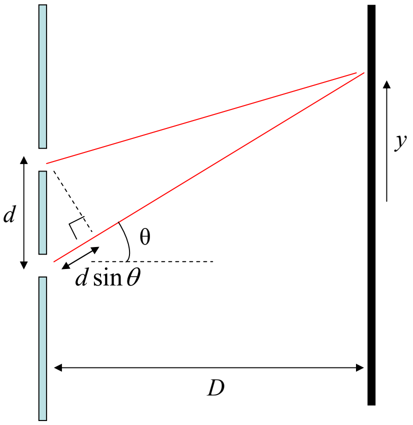
**Experiment:** He atoms at 83K, with *d*=8μm and *D*=64cm

**Measured separation:** $\Delta y = 8.2\,\mu m$

**Predicted de Broglie wavelength:**

$$K = \frac{3kT}{2}, \quad \text{Mass} = 4\text{m}_\text{u}$$

$$\lambda = \frac{h}{\sqrt{3MkT}} = 1.03\times10^{-10}\,\text{m}$$



**Predicted separation:** $\Delta y = 8.4 \pm 0.8\,\mu m$

Good agreement with experiment

## FRINGE SPACING IN DOUBLE-SLIT EXPERIMENT

**Maxima when:** $d\sin\theta = n\lambda$

$D \gg d$ so use small angle approximation

$$\theta \approx \frac{n\lambda}{d}$$

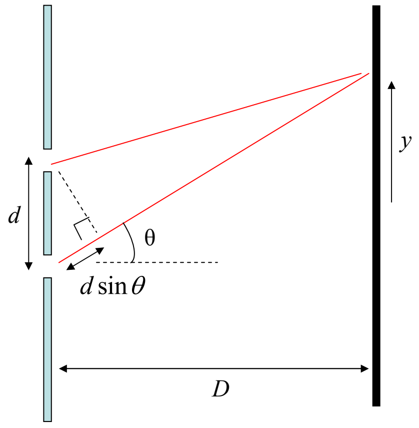
$$\Rightarrow \Delta\theta \approx \frac{\lambda}{d}$$

**Position on screen:** $y = D\tan\theta \approx D\theta$

**So separation between adjacent maxima:**

$$\Delta y \approx D\Delta\theta$$

$$\Rightarrow \Delta y = \frac{\lambda D}{d}$$



## DOUBLE-SLIT EXPERIMENT INTERPRETATION

- The flux of particles arriving at the slits can be reduced so that only one particle arrives at a time. Interference fringes are still observed!

  **Wave-behaviour can be shown by a single atom.**

  **Each particle goes through both slits at once.**

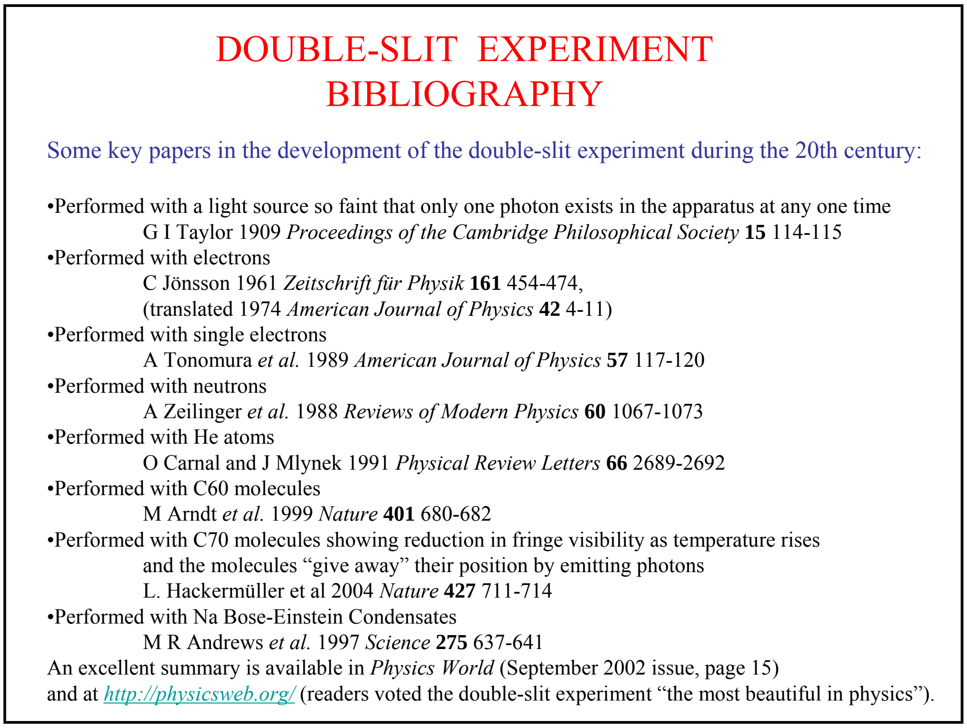
  **A matter wave can interfere with itself**.

  Hence matter-waves are distinct from $H_2O$ molecules collectively giving rise to water waves.
- Wavelength of matter wave unconnected to any internal size of particle. Instead it is determined by the momentum.
- If we try to find out which slit the particle goes through the interference pattern vanishes!

  **We cannot see the wave/particle nature at the same time**.

  If we know which path the particle takes, we lose the fringes .

The importance of the two-slit experiment has been memorably summarized by Richard Feynman: "…a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the *only* mystery."

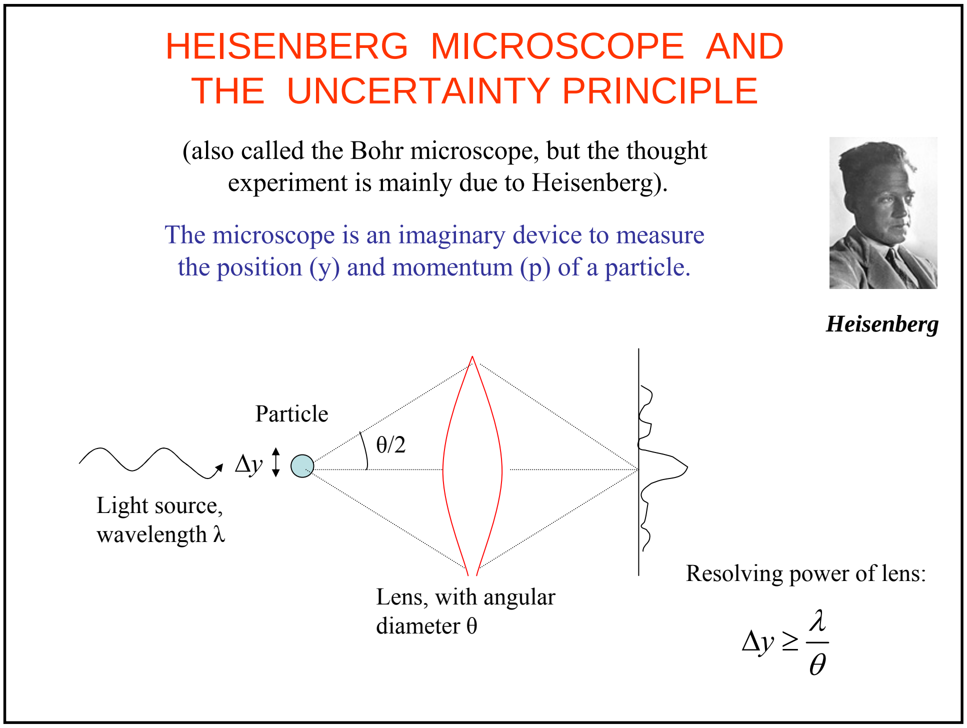


## DOUBLE-SLIT EXPERIMENT BIBLIOGRAPHY

Some key papers in the development of the double-slit experiment during the 20th century:

- Performed with a light source so faint that only one photon exists in the apparatus at any one time
  G I Taylor 1909 *Proceedings of the Cambridge Philosophical Society* **15** 114-115
- Performed with electrons
  C Jönsson 1961 *Zeitschrift für Physik* **161** 454-474,
  (translated 1974 *American Journal of Physics* **42** 4-11)
- Performed with single electrons
  A Tonomura *et al.* 1989 *American Journal of Physics* **57** 117-120
- Performed with neutrons
  A Zeilinger *et al.* 1988 *Reviews of Modern Physics* **60** 1067-1073
- Performed with He atoms
  O Carnal and J Mlynek 1991 *Physical Review Letters* **66** 2689-2692
- Performed with C60 molecules
  M Arndt *et al.* 1999 *Nature* **401** 680-682
- Performed with C70 molecules showing reduction in fringe visibility as temperature rises and the molecules “give away” their position by emitting photons
  L. Hackermüller et al 2004 *Nature* **427** 711-714
- Performed with Na Bose-Einstein Condensates
  M R Andrews *et al.* 1997 *Science* **275** 637-641

An excellent summary is available in *Physics World* (September 2002 issue, page 15) and at http://physicsweb.org/ (readers voted the double-slit experiment “the most beautiful in physics”).



## HEISENBERG MICROSCOPE AND THE UNCERTAINTY PRINCIPLE

(also called the Bohr microscope, but the thought experiment is mainly due to Heisenberg).

The microscope is an imaginary device to measure the position (y) and momentum (p) of a particle.

***Heisenberg***

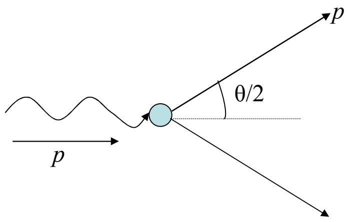
Resolving power of lens:

$$\Delta y \geq \frac{\lambda}{\theta}$$

## HEISENBERG MICROSCOPE (cont)

Photons transfer momentum to the particle when they scatter.

Magnitude of p is the same before and after the collision. Why?



Uncertainty in *photon* y-momentum
= Uncertainty in *particle* y-momentum

$$\boxed{-p\sin(\theta/2) \le p_y \le p\sin(\theta/2)}$$

Small angle approximation

$$\Delta p_y = 2p\sin(\theta/2) \approx p\theta$$

de Broglie relation gives $p = h/\lambda$ and so $\Delta p_y \approx \dfrac{h\theta}{\lambda}$

From before $\Delta y \ge \dfrac{\lambda}{\theta}$ hence $\boxed{\Delta p_y \Delta y \approx h}$

HEISENBERG UNCERTAINTY PRINCIPLE.

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## Point for discussion

The thought experiment seems to imply that, while prior to experiment we have well defined values, it is the act of measurement which introduces the uncertainty by disturbing the particle's position and momentum.

Nowadays it is more widely accepted that quantum uncertainty (lack of determinism) is intrinsic to the theory.

## HEISENBERG UNCERTAINTY PRINCIPLE

We will show formally (section 4)

$$\boxed{\begin{aligned}\Delta x \Delta p_x &\geq \hbar / 2 \\ \Delta y \Delta p_y &\geq \hbar / 2 \\ \Delta z \Delta p_z &\geq \hbar / 2\end{aligned}}$$

HEISENBERG UNCERTAINTY PRINCIPLE.

We cannot have simultaneous knowledge
of 'conjugate' variables such as position and momenta.

Note, however, $\boxed{\Delta x \Delta p_y \geq 0}$ etc

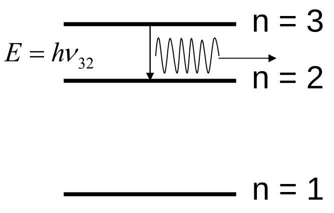
Arbitary precision is possible in principle for
position in one direction and momentum in another

## HEISENBERG UNCERTAINTY PRINCIPLE

There is also an energy-time uncertainty relation

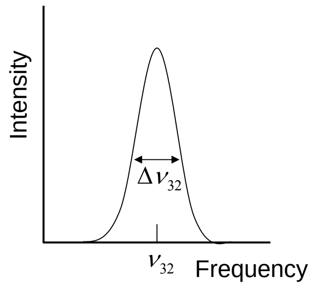
$$\boxed{\Delta E \Delta t \geq \hbar / 2}$$

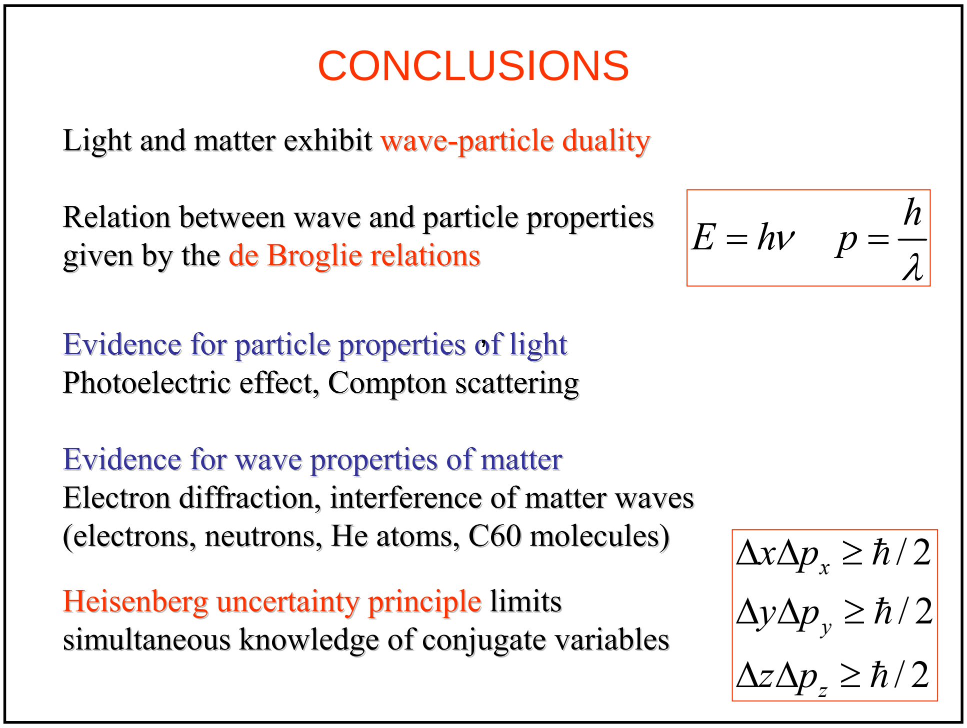
Transitions between energy levels of atoms are not perfectly
sharp in frequency.



An electron in *n* = 3 will spontaneously decay to a lower level after a lifetime of order $t \sim 10^{-8}\,\mathrm{s}$

There is a corresponding 'spread' in the emitted frequency





## CONCLUSIONS

Light and matter exhibit wave-particle duality

Relation between wave and particle properties given by the de Broglie relations

$$E = h\nu \qquad p = \frac{h}{\lambda}$$

Evidence for particle properties of light
Photoelectric effect, Compton scattering

Evidence for wave properties of matter
Electron diffraction, interference of matter waves (electrons, neutrons, He atoms, C60 molecules)

Heisenberg uncertainty principle limits simultaneous knowledge of conjugate variables

$$\Delta x \Delta p_x \geq \hbar / 2$$
$$\Delta y \Delta p_y \geq \hbar / 2$$
$$\Delta z \Delta p_z \geq \hbar / 2$$